



Cairo University

GEAR DESIGN OPTIMIZATION
USING
GENETIC ALGORITHM

By

Ayman Fouad Ibrahim Gemaii

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Mechanical Design and Production Engineering

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
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Gear design optimization using genetic algorithm.

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Summary:

Gears are one of the most important mechanical devices used for power transmission. Gear design optimization holds a wide concern in mechanical design for cost reduction and better performance. Gear design is a complex optimization problem; as it contains continuous, discrete, and integer design variables, rather than large number of design constraints making conventional optimization methods, depending on functions derivations and an initial starting point, inapplicable to solve such problems. In this thesis, GA is used, in addition to MATLAB as an optimization tool, to solve optimization gears design problems to minimize the volume of helical gear train, and to minimize worm gear power losses. Optimization design codes are written and final results are showed and discussed to show GA implementation effect on gear design.

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Table of Contents

| | |
|---|-------------|
| Acknowledgments | I |
| Table of Contents..... | II |
| List of Tables | V |
| List of Figures..... | VI |
| Nomenclature | VIII |
| Abstract..... | X |
| Chapter 1: Introduction | 1 |
| 1.1. Introduction | 1 |
| 1.2. Literature review | 2 |
| 1.2.1. Types of gears | 2 |
| 1.2.2. Optimization process and techniques..... | 5 |
| 1.2.3. Pervious gear optimization works and objectives..... | 8 |
| 1.3. MATLAB..... | 10 |
| 1.4. Work objectives and thesis content..... | 12 |
| Chapter 2: Optimization Process | 12 |
| 2.1. Introduction | 12 |
| 2.2. Optimization definitions and applications | 12 |
| 2.3. Optimization formulation and procedure | 14 |
| 2.4. Optimization concepts..... | 21 |
| 2.4.1. Local and global minima..... | 21 |
| 2.4.2. Optimal conditions..... | 22 |
| 2.5. Modern optimization techniques..... | 25 |
| 2.6. Genetic Algorithm (GA) | 25 |
| 2.6.1 GA terminology | 26 |
| 2.6.2. Design variables representation | 26 |
| 2.6.3. GA operators..... | 28 |
| 2.6.4. GA constraints handling..... | 33 |
| 2.6.5. GA procedure, advantages, and applications | 35 |
| Chapter 3: Gear Design..... | 40 |
| 3.1. Introduction | 40 |

| | |
|--|-----------|
| 3.2. Helical gears | 40 |
| 3.2.1. Helical gears terminology | 42 |
| 3.2.2. Helical gears stresses analysis and materials | 45 |
| 3.3. Worm gears | 54 |
| 3.3.1. Worm types | 55 |
| 3.3.2. Worm gears terminology | 58 |
| 3.3.3. Worm gears force analysis..... | 60 |
| 3.3.4. Worm gear contact stress | 62 |
| 3.3.5. Worm gear bending stress..... | 63 |
| 3.3.6. Worm shaft deflection..... | 63 |
| Chapter 4: Gear Design Optimization | 64 |
| 4.1. Introduction | 64 |
| 4.2. Optimum design process | 64 |
| 4.3. Helical gears design optimization | 67 |
| 4.3.1. Problem description | 67 |
| 4.3.2. Data and information collection..... | 67 |
| 4.3.3. Design variables..... | 68 |
| 4.3.4. Optimization criterion..... | 69 |
| 4.3.5. Constraints formulation..... | 70 |
| 4.3.6. Optimization code and algorithm setting | 73 |
| 4.3.7. Optimization Results..... | 80 |
| 4.4. Worm gear design optimization | 82 |
| 4.4.1. Problem description | 82 |
| 4.4.2. Data and information collection..... | 82 |
| 4.4.3. Design variables..... | 83 |
| 4.4.4. Optimization criterion..... | 83 |
| 4.4.5. Constraints formulation..... | 85 |
| 4.4.6. Optimization code and algorithm setting | 86 |
| 4.4.7. Optimization results | 89 |
| Chapter 5: Results and Discussion | 92 |
| 5.1. Introduction | 92 |
| 5.2. Worm gear sensitivity analysis | 92 |

| | |
|--|------------|
| 5.3. Multi objective function effect..... | 100 |
| 5.4. Pervious work comparison..... | 102 |
| Chapter 6: Conclusion..... | 104 |
| References..... | 106 |
| Appendix A: Improving GA Efficiency | 110 |
| A.1. Parallelization..... | 110 |
| A.2. Hybridization..... | 111 |
| A.3. Time continuation..... | 111 |
| A.4. Evaluation relaxation..... | 111 |
| Appendix B: GA Toolbox..... | 112 |

List of Tables

| | |
|--|-----|
| Table 3.1 Overload factor K_o | 48 |
| Table 3.2 Size factor K_s | 48 |
| Table 3.3 Load distribution factor K_m | 48 |
| Table 3.4 Normal pressure angle α_n | 60 |
| Table 4.1 Actual gear train specifications..... | 68 |
| Table 4.2 Helical gear design variables..... | 68 |
| Table 4.3 Helical gears optimization results..... | 80 |
| Table 4.4 Worm gear data..... | 82 |
| Table 4.5 Worm gear design variables..... | 83 |
| Table 4.6 Analytical results..... | 85 |
| Table 4.7 Worm gear optimization results..... | 89 |
| Table 5.1 Different wheel materials..... | 97 |
| Table 5.2 Helical gear optimization results with multi objective function..... | 101 |
| Table 5.3 Worm gear optimization results comparison..... | 102 |
| Table 5.4 Helical gear optimization results comparison..... | 103 |

List of Figures

| | |
|--|----|
| Figure 1.1: Compound gear train | 2 |
| Figure 1.2: Types of gears | 3 |
| Figure 1.3: (a) Non-circular, (b) Intermittent, (c) Pin gearing, (d) Planetary gear..... | 4 |
| Figure 1.4: Classification of optimization methods | 6 |
| Figure 2.1: Minimum and maximum value of $f(x)$ | 13 |
| Figure 2.2: Effect of multiplication and addition on optimum solution..... | 13 |
| Figure 2.3: (a) Continuous function, (b, c, and d) Discontinuous function | 15 |
| Figure 2.4: Constraint surfaces in a two-dimensional design space | 17 |
| Figure 2.5: Geometry optimization (size, shape, and topology) | 20 |
| Figure 2.6: (a) Local minima, (b) Global minima..... | 21 |
| Figure 2.7: Roulette wheel selection | 29 |
| Figure 2.8: (A) Roulette and (B) Rank selection | 29 |
| Figure 2.9: Tournament selection..... | 30 |
| Figure 2.10: Feasible and infeasible points | 33 |
| Figure 2.11: (a) Exterior and (b) Interior penalty methods | 34 |
| Figure 2.12: GA outline | 36 |
| Figure 2.13: GA procedure..... | 37 |
| Figure 3.1: Helical gear | 40 |
| Figure 3.2: Normal and transverse sections | 41 |
| Figure 3.3: Helix angle direction..... | 42 |
| Figure 3.4: Face advance..... | 43 |
| Figure 3.5: Helical gear geometry..... | 45 |
| Figure 3.6: Force analysis | 46 |
| Figure 3.7: Contact stresses..... | 47 |
| Figure 3.8: Stress cycle factor Z_N | 49 |
| Figure 3.9: Hardness ratio factor CH | 50 |
| Figure 3.10: Gear tooth forces..... | 51 |
| Figure 3.11: Rim thickness factor KB | 52 |

| | |
|---|-----|
| Figure 3.12: Bending geometry factor | 52 |
| Figure 3.13: Stress cycle factor YN | 53 |
| Figure 3.14: Worm gear drive | 55 |
| Figure 3.15: Right and left handed worms | 55 |
| Figure 3.16: Type of worms | 56 |
| Figure 3.17: CAVEX worm characteristics | 57 |
| Figure 3.18: Single-enveloping worm wheel | 57 |
| Figure 3.19: Double-enveloping worm wheel..... | 58 |
| Figure 3.20: Worm axial section | 59 |
| Figure 3.21: Worm gear forces | 61 |
| Figure 3.22: Form factor Z..... | 62 |
| Figure 4.1: Design procedure | 65 |
| Figure 4.2: (a) Conventional design method, (b) Optimum design method | 66 |
| Figure 4.3: Initial helical gears design | 67 |
| Figure 4.4: Helical gear best/mean generation function value..... | 81 |
| Figure 4.5: Initial (left) and optimized (right) helical gears..... | 81 |
| Figure 4.6: Worm gear best/mean generation function value | 90 |
| Figure 5.1: Effect of worm teeth and axial module on power losses | 92 |
| Figure 5.2: Effect of input power and axial module on power losses | 93 |
| Figure 5.3: Effect of input power and worm teeth on power losses..... | 94 |
| Figure 5.4: Effect of pressure angle on power losses..... | 95 |
| Figure 5.5: Effect of pressure angle and input power on power losses..... | 96 |
| Figure 5.6: Effect of pressure angle and axial module on power losses | 97 |
| Figure 5.7: Effect of wheel material on power losses | 98 |
| Figure 5.8: Effect of input speed on power losses | 99 |
| Figure 5.9: Friction coefficient effect on power losses | 100 |
| Figure A.1: Simple master-slave Parallel GA | 110 |
| Figure A.2: Different search methods | 111 |
| Figure B.1: Rank scaling (a, b) before & after ranking respectively | 112 |
| Figure B.2: Stochastic uniform selection | 113 |

Nomenclature

| Symbol | Unit | Definition |
|-------------|-----------------|---|
| a | mm | Gears center distance |
| b | mm | Gear width |
| C_f | -- | Helical gear surface condition factor |
| CR_{FA} | -- | Helical gear face advance contact ratio |
| C_H | -- | Helical gear hardness ratio factor |
| C_p | MPa | Elastic coefficient |
| CR | -- | Helical gear contact ratio |
| CR_T | -- | Helical gear transverse contact ratio |
| d | mm | Gear pitch circle diameter |
| E | MPa | Modulus of elasticity |
| f | mm | Worm shaft deflection |
| F_a | N | Axial force |
| F_A | mm | Helical gear face advance |
| F_r | N | Radial force |
| F_R | N | Worm resultant force |
| F_t | N | Tangential force |
| F_{t2max} | N | Wheel maximum tangential force |
| g_i | -- | Inequality constraint |
| h | -- | Penalty coefficient |
| h_a | mm | Tooth addendum |
| h_f | mm | Tooth dedendum |
| h_j | -- | Equality constraint |
| i | -- | Overall gear ratio |
| I | -- | Helical gear pitting resistance geometry factor |
| I | mm ⁴ | Worm shaft moment of inertia |
| J | -- | Helical gear bending resistance geometry factor |
| K_B | -- | Helical gear rim thickness factor |
| K_m | -- | Helical gear load distribution factor |
| K_o | -- | Helical gear overload factor |
| K_R | -- | Helical gear reliability factor |
| K_s | -- | Helical gear size factor |
| K_T | -- | Helical gear temperature factor |
| K_v | -- | Helical gear dynamic factor |
| L_w | mm | Worm shaft bearing separating distance |
| m | mm | Axial module |
| m_n | mm | Normal module |
| m_N | -- | Helical gear load shearing ratio |

| | | |
|-----------------|-------------------|--|
| m_t | mm | Transverse module |
| n | rpm | Rotational speed |
| N | Cycle | Number of cycles before helical gear tooth breakage |
| N_i | Cycle | Helical gear crack initiation cycles |
| N_p | Cycle | Helical gear crack penetration cycles |
| P | N.m | Torque |
| P | MPa | Contact stress |
| P_a | mm | Worm axial pitch |
| P_{bt} | mm | Helical gear transverse base pitch |
| P_i | Watt | Input power |
| P_t | mm | Helical gear transverse circular pitch |
| P_z | mm | Worm lead |
| P_2 | mm | Output power |
| r_a | mm | Helical gear tip circle radius |
| r_b | mm | Helical gear base circle radius |
| S_F | -- | Helical gear bending stress safety factor |
| S_H | -- | Helical gear contact stress safety factor |
| S_q | mm | Helical gear tooth maximum bending stress position width |
| t | -- | Helical gear teeth |
| T, M_t | N.m | Torque |
| V | m/sec | Gear pitch line velocity |
| V_{max} | m/sec | Maximum gear pitch line velocity |
| y | -- | Helical gear penalty function |
| Y_N | -- | Helical gear bending resistance stress cycle factor |
| z | mm | Helical gear action line length |
| Z_N | -- | Helical gear pitting resistance stress cycle factor |
| Z_p | -- | Worm gear proportional factor |
| z_W | -- | Wheel teeth |
| Z_z | -- | Worm gear form factor |
| σ_b | MPa | Bending stress |
| σ_{ball} | MPa | Allowable bending stress |
| σ_H | MPa | Contact stress |
| σ_{Hall} | MPa | Allowable contact stress |
| α | Degree | Pressure angle |
| α | -- | Weight factor |
| α_a | Degree | Axial pressure angle |
| α_t | Degree | Transverse pressure angle |
| γ | Degree | Worm lead angle |
| β | Degree | Helical gear helix angle |
| ω | sec^{-1} | Angular speed |
| ν | -- | Poisson's ratio |
| μ | -- | Coefficient of friction |
| η | -- | Efficiency |

Abstract

Gears are one of the oldest mechanical elements used for motion and torque transmission between rotating shafts, and is almost used in all mechanical devices. Designing gears, as well as other mechanical components, for maximum efficiency and operating life time, minimum volume and weight, or combination between them plays an important trend in mechanical design for material costs reduction, ease to manufacturing, space saving, smooth running, market competition, and legal and environmental restrictions. Optimization is a field of science which seeks the solution that makes usage of available design resources in an efficient and effective manner leading to the superior acquisition of these resources, and supports taking decisions in a scientific and logical manner. Traditional optimization techniques are based on mathematical, statistical, and numerical approaches, while non-traditional (modern) optimization techniques are originated based on biological and natural aspects. Genetic Algorithm GA is a modern optimization technique which mimics mechanics of natural genetics and selection based on Darwin's theory of natural selection and survival of the fittest.

The present work is an application of Genetic Algorithm optimization technique to solve optimum design of gears. The design methodology, preliminary and detailed design, is considered. A two stage helical gears parameters; module, gear face width, helix angle, and number of teeth are designed and optimized to reduce the gears volume through MATLABR2013b GA optimization toolbox. The gear parameters are confined by bending and contact stresses, rather than geometrical and shape constraints. A worm gear drive is also optimized to reduce power losses; through adjusting gear number of teeth, coefficient of friction, and worm lead angle, to satisfy stresses constraints. Pervious objectives are studied through single and multi-objective functions optimization.

Optimum design results and the sensitivity of the design problem to different input parameters are shown by using design curves. A comparison is made between present optimization results and the previous results showing improving in worm gear power losses and efficiency, while same optimization results are obtained in helical gears. The results of the current work show that GA is a comprehensive and powerful optimization technique applicable to solve complex design problems effectively and has a high probability of global points.

Chapter 1: Introduction

1.1. Introduction

Design is one of the most comprehensive of human activity, and is defined as a plan for arranging elements to achieve a specific purpose. Design can be also defined as a strategy specifying requirements, parameters, financials, tasks, and operations which fulfill legal, political, environmental, and safety restraints to achieve someone expectations. So, design is of a major concern and is almost used in all fields as graphics, web sites, software coding, fashion design...etc., rather than engineering discipline. Design theory is one of design fields in which the design process is the main goal and scope of such concern. The aim of the design theory is to develop models, using all data and available information about the design, for thoroughly understanding the design case; by studying its behavior to different parameters and experiments, and setting-up tools which uphold the design, to finally obtain detailed instructions for the design objective illustrated in an artificial language. Among design disciplines is mechanical design which get utilized of engineering, physics, and material science to specify different mechanical systems dimensions and shapes, which requires cooperation between different engineering departments [8, 9, 11].

Several solutions may be found for a design problem, whilst the solution that makes usage of available design resources in the most efficient and effective manner leading to the superior acquisition of these resources is an optimum solution for the design problem. Thus; optimization is a field of science which seeks an approach of best alternatives from available alternatives under certain circumstances. Lack of manufacturing materials, lowering costs, and improving products performance and efficiency are fields of optimization usage in engineering design [4,12].

Gears are one of the oldest mechanical elements used for motion and torque transmission between rotating shafts, and is almost used in all mechanical devices. Gears usage dates back to 4th century BCE in China, and the Greek mechanics (287-212 BCE). They were used in water and wind mills different applications, direction-pointing mechanisms which have been developed to calculate astronomical positions, mechanical clocks in the year 1386. Gears were developed in a slow rate until the raise of the first industrial revolution (starting 1760) and the usage of steam power. Further development, till now, of gears took place during the second industrial revolution (1840-1870) and the rapid development of machinery. Gears have the advantage over other power transmission devices, as chain-sprocket and belt-pulley drive, of operating at higher speeds and transmitting higher loads with precise velocity ratio, high efficiency, reliability, and service life, and their compact design applicable for small devices like watches; on the other hand, gears have higher manufacturing and operating costs[8].

The recently trend of designing more complicated, high-tech, and advanced engineering products, including different types of gears, in short times, for technology progress increases the