



Ain Shams University

Numerical Treatment Based on Spectral Methods for Diffusion Like Problems

Thesis by

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Submitted To

Department of mathematics - Faculty of Science

Ain Shams University - Cairo - Egypt

for the Degree of Doctor of Philosophy

in Pure Mathematics

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Dedication

DEDICATION

To

All members of my family

They are the main source of my success.

With special gratitude to my wife, for her
unwavering support, patience and loving care.

She was a gift from the beginning.

To my children, they make it worth.

To all who they have a special place in my heart.

With mercy and blessings of Allah



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In the name of Allah, the most merciful, the most compassionate all praise be to Allah, the Lord of the worlds; and prayers and peace be upon his Holy Prophet Mohamed (God bless and give him Peace) and his very noble Companions and his Followers (blessing and peace be upon them) and who they have faith and practice of genuine Islam after them.

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Adel Rashed abd Ali

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Abstract

ABSTRACT

Thesis Title: Numerical Treatment Based on Spectral Methods for Diffusion Like Problems

Diffusion like problems appear as developments of the classical diffusion problems through the appearance of fractional order differential terms or integral terms. The numerical treatment for such problems is one the important subjects due to the difficulty in obtaining analytical solutions. The thesis considers the differences between the models contain only differential terms and models containing integral terms or fractional order derivatives, also the thesis introduces a numerical treatment to diffusion like problems that contain fractional order derivatives in addition to integral terms.

The thesis discusses numerical treatments by using the finite difference method for models that contain fractional order derivatives. Also, extensions of multilevel treatments of the classical models are taken into account in the treatment of fractional order case. As a remedy to large amount of calculations and difficulties in the two dimensional models the alternating direction implicit method has been used. The Haar wavelets technique is used in solving the Diffusion like equation. Also, the inverse operator concept which extend the effect of the memory and hereditary properties to the spatial terms. Numerical examples with graphical representation of the results are given to support the theoretical results.



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