A STUDY OF THE OPERATING PARAMETERS OF CHAOTIC SEMICONDUCTOR LASERS

Thesis

Submitted to
The National Institute of Laser Enhanced Sciences
(NILES) - Cairo University
In Partial Fulfillment of the Requirements
For the Degree
of
Master of Science

In **Engineering Applications of Lasers**

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Approval Sheet

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