UNIVERSITY OF MIAMI

THE TITLE FOR MY UNIVERSITY OF MIAMI THESIS

By

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A THESIS
Submitted to the Faculty
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in partial fulfillment of the requirements for
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A Thesis submitted in partial fulfillment of
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Chapter 1

Basic Concepts

In this chapter we will review briefly the basic geometric, topological and algebraic concepts and results relevant to this work.We adapted these concepts to fit coherently into the framework of this thesis.

1.1 Manifold:

Manifolds are spaces in which the environment of each point is "just like" a small piece of an Euclidean space.The most familiar examples of manifolds are smooth surfaces like the sphere or the torus , where each point lies in a little curved disc that may gently flattened into a disc in the plane . But the cone is not qualified as a manifold since no nieghbourhood of the vertex point like a simple piece of the plane . See Fig. (1.1) .

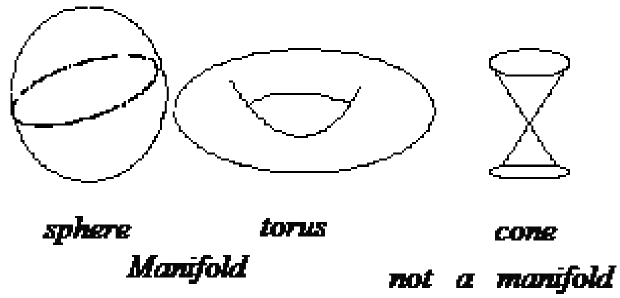

Fig.(1.1)

To give the mathematical definition , we need first to give some notation .

Let R be the set of real numbers . For an integer $n > 0$, let R^n be the product space of ordered $n-$tuples of real numbers.Thus $R^n = [(a_1, \ldots, a_n) : a_i \text{ in } R]$.

For $i = 1, \ldots, n$, let u_i be the natural coordinate (slot) functions of R^n. $i.e.$ $u_i : R^n \to R$ by $u_i(a_1, \ldots, a_n) = a_i$.

An open set of R^n will be a set which is open in the standard metric topology induced by the standard metric function d on R^n thus if $a = (a_1, \ldots, a_n)$ and $b = (b_1, \ldots, b_n)$ are points in R^n, then

$$d(a, b) = [\sum_{i=1}^{n} (a_i - b_i)^2]^{\frac{1}{2}}.$$

The concept of differentiability is based ultimately on the definition of a derivative in elementary calculus .

Let r be an integer ,$r > 0$.Recall from advanced calculus that a map f from an open set $A \subset R^n$ into R is called C^r on A if it possesses continuous partial derivatives on A of all orders $\leq r$.

If f is merely continuous from A to R, then f is C^0 on A .

If f is C^ron A for all r,then f is C^∞on A . If f is real analytic on A (expandable in a power series in the coordinate functions about each point of A) ,then f is C^w on A. Henceforth , unless otherwise specified , we let r be ∞, w,or an integer > 0.

A map f from an open set $A \subset R^n$ into R^k(k an integer≥ 1) is C^r on A if each of its slot functions $f_i = u_i o f$ is C^ron A for $i = 1, \ldots, k$; thus for p in R^n, $f(p) = (f_1(p), \ldots, f_k(p))$ in R^k.

We now define a manifold . Let M be a set . An $n-$coordinate pair on M is a pair (Φ, U) consisting of a subset U of M and 1 to 1 map Φ of U onto an open set in R^n. One $n-$coordinate pair (Φ, U) is C^r related to another $n-$coordinate pair (θ, V) if the maps $\Phi \circ \theta^{-1}$ and $\theta \circ \Phi^{-1}$ are C^r maps wherever they are defined (thus their domains of definition must be open) . A C^r $n-$ subatlas on M is a collection of $n-$coordinate pairs (Φ_n, U_n), each of which is C^r related to every other member of the collection ,and the union of the sets U_n is M .See Fig.(1.2) .

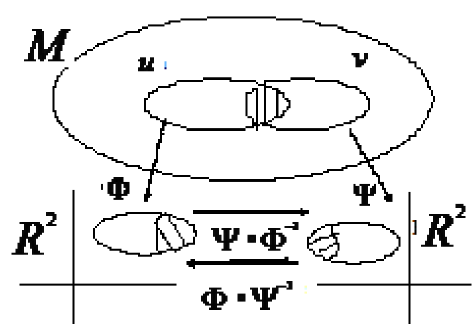

Fig.(1.2)

A maximal collection of C^r related $n-$coordinate pairs is called a C^r $n-$atlas .If a C^r $n-$atlas contains a C^r $n-$subatlas,we say the subatlas induces or generates the atlas .

Finally , an $n-$dimensional C^r manifold or a C^r $n-$manifold is a set M together with a C^r $n-$atlas.

When $r = 0$, M is customarily called a locally Euclidean space or a topological manifold , and only when $r \neq 0$ M is called a differentiable or smooth manifold . An atlas on a set M is often called a differentiable structure or a manifold structure[3,6,9] .

1.1.1 Smooth Maps :

a) A map $f : U \to R^m$ from an open set $U \subset R^n$ into $R^m$is called smooth if it has continuous derivatives of all orders .

b) A map $f : X \to R^m$ from an arbitrary subset X in R^n into R^m is called smooth if it may be locally extended to a smooth map on open sets .i.e. for each point $x \in X$ there is an open set $U \subset R^n$ and a smooth map $F : U \to R^m$ such that

$f(x) = F(x)$, $x \in U \cap X$. See Fig.(1.3) .

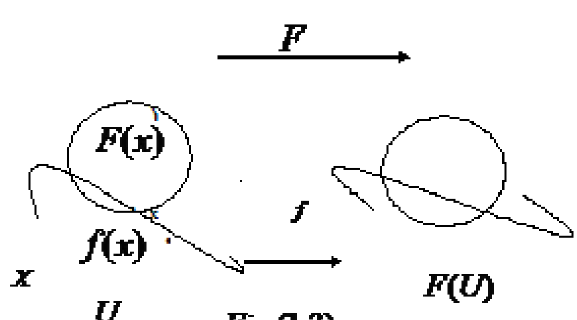

Fig.(1.3)

1.1.2 Smooth Manifolds:

Let M be a non empty (second – countable - Hausdorff) topological space such that :

1) M is the union of open subsets U_α, and each U_α is equipped with a homeomorphism Φ_α , taking U_α to an open set in R^n.

i.e.

$\Phi_\alpha : U_\alpha \to \Phi_\alpha \quad U_\alpha \subset R^n$

2) If $U_\alpha \cap\ U_\beta \neq \varnothing$ then the overlapped map

$\Phi_\beta\ \Phi_\alpha^{-1} : \Phi_\alpha\ (U_\alpha \cap\ U_\beta) \to \Phi_\beta(U_\alpha \cap\ U_\beta)$

is a smooth map .

Each pair $(U_\alpha, \Phi_\alpha)$is called a chart on M ,and the collection $A = \{(U_\alpha, \Phi_\alpha)\}$ of all charts is called (smooth) atlas on M.

The space M taken together with the atlas A will be called a smooth manifold of dimension n or smooth $n-$manifold or $C^\infty n-$manifold .

If the space M satisfies condition (1) only themanifold will be called a topological manifold orsometimes just a manifold [9].

See Fig.(1.4) .

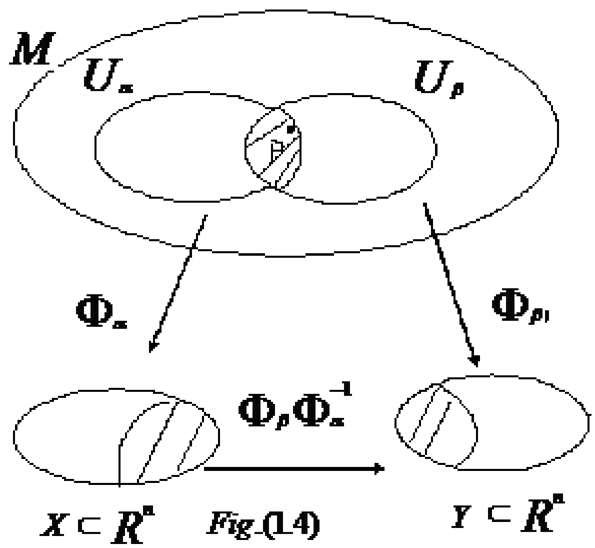

Fig.(1.4)

1.2 Group :

In mathematics, a group is an algebraic structure consisting of a set together with an operation that combines any two of its elements to form a third element. To qualify as a group, the set and the operation must satisfy a few conditions called group axioms, namely closure, associativity, identity and invertibility.Many familiar mathematical structures such as number systems obey these axioms: for example, the integers endowed with the addition operation form a group. However, the abstract formalization of the group axioms, detached as it is from the concrete nature of any particular group and its operation, allows entities with highly diverse mathematical origins in abstract algebra and beyond to be handled in a flexible way, while retaining their essential structural aspects. The ubiquity of groups in numerous areas within and outside mathematics makes them a central organizing principle of contemporary mathematics[7].

1.2.1 Definition:

If G is a set and $*$ is a binary operation then the system $(G, *)$ is called a group if the following conditions are satisfied :

$1 - \forall\ a, b \in G \Rightarrow (a * b) \in G$

$2-\ If\ a, b, c \in G \Rightarrow (a * b) * c = a * (b * c)$

$3-\ a * e = e * a = a \quad for\ all \quad a \in G$

$4-\ a * a^{-1} = a^{-1} * a = e\ for\ all \quad a, a^{-1} \in G.$

1.2.2 An abelian group:

An abelian group is a set A, together with a binary operation $+$ defined on A and satisfying the following five axioms:

(1) For all a, b in A, $a + b \in A$.

(2) For all a, b, c in A, $a + (b + c) = (a + b) + c$.

(3) There exists $0 \in A$ such that, for all $a \in A$, $a + 0 = 0 + a = a$.

(4) For each $a \in A$ there exists $b \in A$ such that $a + b = b + a = 0$.

(5) For all a, b in A, $a + b = b + a$.

It follows from these axioms that the 0 of (3) is unique and that , in (4), b is uniquely determined by a . We write $-a$ for this unique b and abbreviate $c + (-a)$ to $c - a$ for any $c \in A$ [8].

1.2.3 A sub group:

A subset of A which is also an abelian group under the same binary operation. Thus $B \subset A$ is a sub group of A if and only if B is nonempty and, for all $a, b \in B$, we have $a - b \in B$ [8].

1.2.4 Zero group:

Any group with precisely one element is denoted 0 and called a trivial or zero group[8].

1.2.5 Order group:

Let $a \in A$. If, for all $n \in \mathbb{Z}, n \neq 0$, we have $na \neq 0$ then a is said to have infinite order. Other wise,the order of a is the smallest integer $n > 0$ such that $na = 0$. Thus a has order 1 if and only if $a = 0$ and, for any a ,a and $-a$ have the same order, or both have infinite order.

The order $|A|$ of a group A is simply the number of elements in A[8].

1.2.6 Cyclic group:

A is called cyclic if there exists $a \in A$ such that any $b \in A$ is of the form na for some $n \in \mathbb{Z}$. Such an a is called a generator of A. Note that $-a$ is also a generatore[8].

1.2.6.1 Properties:

Given a cyclic group G of order n (n may be infinity) and for every g in G,

(1)G is abelian; that is, their group operation is commutative: $gh = hg$ (for all h in G). This is so since $g + h \bmod n = h + g \bmod n$.

(2)If n is finite, then $g^n = g^0$ is the identity element of the group, since $kn \bmod n = 0$ for any integer k.

(3)If $n = \infty$, then there are exactly two elements that generate the group on their own: namely 1 and -1 for $\mathbb{Z}$.

(4)Every subgroup of G is cyclic. Indeed, each finite subgroup of G is a group of $\{0, 1, 2, 3, ...m - 1\}$ with addition modulo m. And each infinite subgroup of G is $m\mathbb{Z}$ for some m, which is bijective to (so isomorphic to) $\mathbb{Z}$.

1.2.6.2 Examples:

(1) The integers $\mathbb{Z}$ under addition is a cyclic group with generator 1 (or , alternatively, -1),which has infinite order.

(2) The set $\{0, 1, ..., k-1\}$ $(k \geq 1)$ under addition modulo k is a cyclic group $\mathbb{Z}_k$ with generator 1 (or, in fact, any element coprime to k), which has order k. Note that $\mathbb{Z}_1 = 0$.

(3) For any A and $a \in A$ the subset $\{na : n \in \mathbb{Z}\}$ is a subgroup of A which is cyclic, a being a generator. It is called the subgroup generated by a and has order equal to the order of a (infinite order if a has infinite order) [8].

1.2.7 Symmetric group:

The symmetric group S_n of degree n is the group of all permutations on n symbols .S_n is therefore a permutation group of order $n!$ and contains as subgroups every group of order n .The th symmetric group is represented in Mathematica as SymmetricGroup[n], and an (inefficient) permutation group representation is given by SymmetricGroup[n] in the Mathematica package Combinatorica‘ [14].

1.2.7.1 Examples:

(1) Let $n = 2$ then $|S_2| = 2! = 2 \times 1 = 2$ then

$S_2 = \{\left\{\begin{smallmatrix} 1 & 2 \\ 1 & 2 \end{smallmatrix}\right\}, \left\{\begin{smallmatrix} 1 & 2 \\ 2 & 1 \end{smallmatrix}\right\}\}$.

(2) Let $n = 3$ then $|S_3| = 3! = 3 \times 2 \times 1 = 6$ then

$S_3 = \{\left\{\begin{smallmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{smallmatrix}\right\}, \left\{\begin{smallmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{smallmatrix}\right\}, \left\{\begin{smallmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{smallmatrix}\right\}, \left\{\begin{smallmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \end{smallmatrix}\right\},$

$\left\{\begin{smallmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{smallmatrix}\right\}, \left\{\begin{smallmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{smallmatrix}\right\}\}$.

1.2.8 Dihedral group:

The dihedral group D_n is the symmetry group of an n-sided regular polygon for $n > 1$. The group order of D_n is $2n$. Dihedral groups D_n are non-Abelian permutation groups for $n > 2$. The th dihedral group is represented in Mathematica as DihedralGroup[n], and an (inefficient) permutation group representation is given by DihedralGroup[n] in the Mathematica package Combinatorica‘ [14] .

1.3 Graph :

There are many physical system whose performance depends not only on the characteristics of the components but also on the relative locations of the elements. An obvious example is an electrical network. If we change a resistor to a capacitor, generally some of the properties (such as an input impedance of the network) also change.This indicates that the performance of a system depends on the characteristics of the components. If, on the other hand, we change the location of one resistor, the input impedance again may change,which shows that the topology of the system is influencing the system’s performance.

There are systems constructed of only one kind of component so that the system's performance depends only on its topology. An example of such a system is a single-contact switching circuit. Similar situations can be seen in nonphysical systems such as structures of administration.
Hence it is important to represent a system so that its topology can be visualized clearly.One simple way of displaing a structure of a system is to draw a diagram consisting of points called "vertices" and line segments called "edges" which connect these vertices so that such vertices and edges indicate components and relationships between these components.
Such a diagram is called a "Linear graph" whose name depends on the kind of physical system we deal with. This means that it may be called a network, a net, a circuit,a graph, a diagram, a structure, and so on.
Instead of indicating the physical strusture of a system, we frequently indicate its mathematical model or its abstract model by a "Linear graph".
Undre such a circumstance, a linear graph is referred to as a flow graph, a signal flow graph, a flow chart, a state diagram, a simplicial complex, a sociogram, an organization diagram, and so forth.
The earliest known paper on linear graph theory, in 1736, is due to "Euler" who gave a solution to the " Konisberger" bridge problem by introducing the concept of linear graphs. In 1847,"Krichhoff" employed linear graph theory for an analysis of electrical networks, known today as the topological formulas for driving point impedance and transfer admittances.This probably is the first paper that applies the theory of linear graphs to engineering pro-

blems. However, it is not "Krichhoff's paper" but "Mobins conjecture", about 1840, concerning the four-color problem that seems to attract many scholars to devote themselves to linear graph theory.

In the past few years, graph theory has established itself as an important mathematical tool in a wide variety of subjects, ranging from operational research and linguistics to chemistry and genetics; at the same time it has also emerged as a worthwhile mathematical discipline in its own right[9].

The graph theory is being applied in many different fields such as engineering system science, social science and humanrelations, business administration and scientific management, political science, physical and organization systems, the electrical circuits and networks, route maps, architectural floor plans, chemistry, ecology, transportation theory, system diagnosis, music,...[8].

1.3.1 Definition:

A graph is intuitively a finite set of points in space, called the vertices of the graph, some pairs of vertices being joined by arcs, called the edges of the graph.

Two arcs are assumed to meet , if at all , in a vertex , and it is also assumed that no edge joins a vertex to itself and that two vertices are never connected by more than one edge[8] .

1.3.2 An abstract graphs:

An abstract graph is a pair (V, E) where V is a finite set and E is a set of unordered pairs of distinct elements of V .Thus an element of E is of the form $\{v, w\}$ where v and w belong to V and $v \neq w$.