Ain Shams University
Faculty of Education
Department of Mathematics

Study of the External Forces Effects on the Flow of Newtonian and non-Newtonian Fluids, with Applications on Bio-fluids.

Thesis

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[بسم الله الرحمن الرحيم]

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Contents

SUMMARY

The flow of Newtonian and non-Newtonian fluids through surfaces or different geometric shapes is playing an important role in the fluid mechanics fields, where it has many important bio-applications in different fields of science, such as biological, chemical, astronomical, geophysical and different industrial applications. The study of the physical effect on the fluid, such as the magnetic fields, the porous media and the presence of various chemical reactions is very important in controlling the velocities, temperature and concentration of the fluid. Therefore, the thesis derives its importance as it deals with some problems in this area.

The thesis consists of four chapters as follows:-

Chapter 1

In this chapter, we presented a general introduction containing the following items:

(1.1) The basic definitions of fluids

(1.2) The difference between Newtonian and non-Newtonian fluids

(1.3) Fundamental equations of motion

(1.4) The heat transfer of the fluid and its different types

(1.5) The mass transfer and its basic equations

(1.6) Magnetohydrodynamics (MHD) and basic equations

(1.7) The flow through porous medium

(1.8) Some numerical methods, which used in our study

(1.9) Applications of fluid flow

Chapter 2

In this chapter, we extend the previous work of Umavathi and Malashetty [45] to study the effect of variable viscosity, variable thermal diffusivity, permeability of the porous medium and a transverse magnetic field with constant strength on the flow of Magnetohydrodynamic (MHD) Newtonian fluid (e.g. air or water) in a vertical channel through a porous medium. The energy equation that includes both viscous and Darcy dissipation terms is presented. The problem is solved analytically for two special cases according to different values of the viscosity variation parameter β_2 (for air and for water). Finally, the numerical solution for the general case was studied numerically by employing the shotting technique. The comparisons were made

between the special cases and the numerical solution of the general case and illustrated graphically to assess the accuracy of our calculations. Also, the present results were compared with the previous results obtained by Umavathi and Malashetty [45] and illustrated graphically and by using tables.

Chapter 3

The purpose of this chapter is to study the effects of the chemical reaction and radiation absorption on the free convective unsteady flow of viscous fluid past a semi-infinite vertical permeable moving plate through a porous medium. The system is expressed by a transverse uniform magnetic field with chemical reaction subjected to a variable suction velocity at the permeable boundary. The problem was solved in two different cases: Firstly, the problem was solved analytically in the absence of the gravitational force when the fluid flowing over a horizontal permeable moving plate. Secondly, the problem was solved numerically for the flow over a vertical permeable moving plate. Comparisons are made between the analytical and numerical solutions, in the absence of the gravitational force, for the velocity, temperature and concentration distributions and illustrated graphically through a set of figures.

Chapter 4

In this chapter, we studied the effect of variation of different physical parameters on the axisymmetric flow of the blood through a stenosed artery by treating the blood in the core region and the plasma in the peripheral layer. The two models (Casson and Biviscosity) for the blood flow were considered to illustrate the agreement between the two models except in the very low shear rate region. The results were illustrated graphically for the axial flow velocity w, pressure gradient $\frac{dp}{dz}$, impedance λ, shear stress force F, and wall shear stress τ_w at the throats and at the critical height by using different values of parameters.

Chapter 1

Introduction to Fluid Mechanics

Introduction to Fluid Mechanics

In this chapter we will introduce some basic concepts of fluid mechanics, which widely used in our thesis. We can illustrate these concepts in the following basic points:

(1.1) The basic definitions of fluids.

(1.2) The difference between Newtonian and non-Newtonian fluids.

(1.3) Fundamental equations of motion.

(1.4) The heat transfer of the fluid and its different types.

(1.5) The mass transfer and its basic equations.

(1.6) Magnetohydrodynamic and basic equations.

(1.7) The flow through porous medium.

(1.8) Some numerical methods, which used in our study.

(1.9) Applications of fluid flow.

Now, we will explain these points in following sections.

(1.1) The basic definitions of fluids:

(1.1.1) Definition of fluid

A fluid is distinguished from a solid by its response to an applied shear stress. Consider a solid body under a shear stress as shown in figure (1.1).

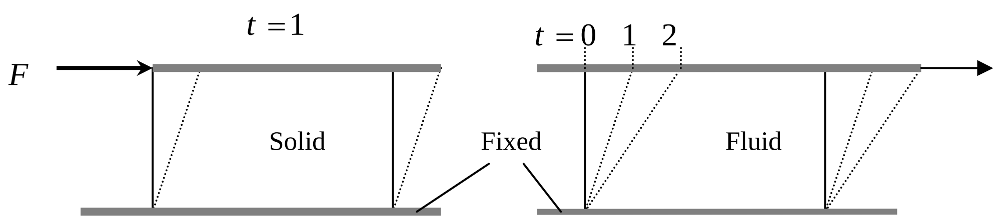

Fig. 1.1

The shear stress introduces deformation, which sets up internal resistance. The deformation grows until the resisting shear equals the applied shear and then stops. On the other hand, such an equilibrium position can be never realized for the fluid, however the applied stress is small. The deformation continues as long as the shear force is maintained. Fluid includes liquid and gases, in liquids molecules are packed closer with significant forces of attraction. As temperature increases, the difference becomes smaller until a liquid gets transformed into a gas. A liquid tends together in globules if taken in small quantities and forms a free surface in large volume. Further, pressure and temperature changes have practically no effect on their volume. For all practical purposes, they can be treated as incompressible. Gases, on the other hand, fill the entire available volume and are readily compressible [1].

(1.1.2) Definition of fluid mechanics

Fluid mechanics is a science that makes use of the basic laws of mechanics and thermodynamics to describe the motion of fluids. Here fluids are understood to be all the media that cannot be assigned clearly to solids, no matter whether their properties can be described by simple or complicated material laws. Gases, liquids and many plastic materials are fluids whose movements are covered by fluid mechanics. Fluids in a state of rest are dealt as a special case of flowing media, i.e. the laws for motionless fluids are deduced in such a way that the velocity in the basic equations of fluid mechanics is set equal to zero. In fluid mechanics, however, one is not content with the formulation of the laws by which fluid movements are described, but makes an effort beyond that to find solutions for flow problems, i.e. for given initial and boundary conditions. To this end, three methods are used in fluid mechanics to solve flow problems:

(a) Analytical solution methods (analytical fluid mechanics): Analytical methods of applied mathematics are used in this field to solve the basic flow equations, taking into account the boundary conditions describing the actual flow problem.

(b) Numerical solution methods (numerical fluid mechanics): Numerical methods of applied mathematics are employed for fluid flow simulations on computers to yield solutions of the basic equations of fluid mechanics.

(c) Experimental solution methods (experimental fluid mechanics): This sub-domain of fluid mechanics uses similarity laws for the transferability of fluid mechanics knowledge from model flow investigations. The knowledge gained in model flows by measurements is transferred by means of the constancy of known characteristic quantities of a flow field to the flow field of actual interest [2].

(1.1.3) Solids and Fluids

All media can be subdivided into solids and fluids, the difference between the two groups being that solids possess elasticity as an important property, whereas fluids have viscosity as a characteristic property. Shear forces imposed on a solid from outside lead to inner elastic shear stresses which prevent irreversible changes of the positions of molecules of the solid. When, in contrast, external shear forces are imposed on fluids, they react with the build-up of velocity gradients, where the build-up of the gradient results via a molecule-dependent momentum transport, i.e. momentum transport through fluid viscosity. Thus elasticity (solids) and viscosity (liquids) are the properties of matter that are employed in fluid mechanics for subdividing media. However, there are a few exceptions to this subdivision, such as in the case of some of the materials in rheology exhibiting mixed properties. They are therefore referred to as visco-elastic media. Some of them behave such that for small deformations they behave like solids and for large deformations they behave like liquids. At this point, attention is drawn to another important fact regarding the characterization of fluid properties. A fluid tries to evade the smallest external shear stresses by starting to flow. Hence it can be inferred from this that a fluid at rest is characterized by a state which is free of external shear stresses. Each area in a fluid at rest is therefore exposed to normal stresses only. When shear stresses occur in a medium at rest, this medium is assigned to solids. The viscous (or the molecular) transports of momentum observed in a fluid, should not be mistaken to be similar to the elastic forces in solids. The viscous forces cannot even be analogously addressed as elastic force. In general, as said above, fluids can be subdivided into liquids and gases. Liquids and some plastic materials show very small expansion coefficients, whereas gases have much larger expansion coefficients. A comparison of the two subgroups of fluids shows that liquids fulfill the condition of incompressibility with a precision that is adequate for the treatment of most flow problems [2].

(1.1.4) Coefficient of viscosity

The viscosity of a fluid is that characteristic of real fluid which exhibits a certain resistance to alteration of form. Viscosity is also known as internal friction. All known fluids [gases or liquids] possess the property of viscosity in varying degrees. The coefficient of viscosity of a fluid may be defined as the tangential force required per unit area to maintain a unit velocity gradient, i.e., to maintain unit relative velocity between two layer unit distance apart. The dimensions of the coefficient of viscosity μ can be found as follows

$$\mu = \frac{\text{Shearing stress}}{\text{Velocity gradient}} = \frac{\text{Force /area}}{\text{Velocity/Length}} = ML^{-1}t^{-1} \ , \tag{1.1}$$

where M , L and t refers to units of mass, length and time respectively.[3]

(1.1.5) Kinematic viscosity

Is the ratio between the coefficient of viscosity μ and the mass density ρ (the mass of unit volume of the fluid, which has the dimensions ML^{-3}).[3]

$$\upsilon = \frac{\mu}{\rho} = \frac{ML^{-1}T^{-1}}{ML^{-3}} = L^2T^{-1} \, . \tag{1.2}$$

(1.1.6) Thermal conductivity

The coefficient of thermal conductivity can be defined from Fourier's law of heat conduction, which supposed that two parallel layers of fluid at a distance d apart, are kept at different temperatures T_1 and T_2 (one of the layers may be solid surface). Fourier noticed that a flow of heat is set up through the layer such that the quantity of heat Q_T transferred through unit area in a unit time is directly proportional to the difference of temperature between the layers and inversely proportional to the distance d , thus he found

$$Q_T = \alpha \frac{T_1 - T_2}{d} \, , \tag{1.3}$$

where α is the constant of proportionality and is known as the coefficient of thermal conductivity. If the distance d between the two layers of fluid is infinitesimal the above law can be written in the differential form as

$$Q_T = -\alpha \frac{dT}{dY} \, , \tag{1.4}$$

which is the Fourier's law of heat conduction, where the negative sign has been taken because the heat flows in the direction of decreasing temperature. [3]

(1.1.7) Coefficient of skin-friction

The dimensionless shearing stress on the surface of a body due to a fluid motion, is known as 'local skin-friction coefficient' and is defined as

$$C_f = \frac{\tau_w}{\rho V^2/2} \tag{1.5}$$

Where τ_w is the local shearing stress on the surface of the body, and $\vec{V}$ is the velocity of the fluid. [3]

(1.1.8) Porosity

The volume fraction occupied by voids, i.e., the total void volume divided by the total volume occupied by the solid matrix and void volumes, is called the porosity. Each void is connected to more than one other pore (interconnected), connected only to one other pore (dead end), or not connected to any other pore (isolated). Fluid flows through the interconnected pores only. The volume fraction of the interconnected pores is called effective porosity. In nonconsolidated media, e.g., particles loosely packed the effective porosity and porosity is equal [4]. So, to introduce a mathematical form of porosity let P be a mathematical point inside the domain occupied by the porous medium, consider a volume ΔU_i [say having the shape of a sphere] much larger than a single pore or grain, for which P is the centroid. For this volume we may determine the ratio

$$n_i \equiv n_i(\Delta U_i) = (\Delta U_v)_i / \Delta U_i \quad , \tag{1.6}$$

where $(\Delta U_v)_i$ is the volume of void space within ΔU_i, repeating the same procedure, a sequence of values $n_i(\Delta U_i), \quad i = 1,2,3$ may be obtained by gradually shrinking the size of ΔU_i around P as a centroid $\Delta U_1 > \Delta U_2 > \Delta U_3 \cdots$. Then the medium's volumetric porosity $n(P)$ at point P is defined as the limit of the ratio n_i as $\Delta U_i \to \Delta U_0$:

$$\mathrm{n(P)} = \lim_{\Delta U_i \to \Delta U_0} \mathrm{n_i}\{\Delta \mathrm{U_i(P)}\} = \lim_{\Delta U_i \to \Delta U_0} \frac{(\Delta \mathrm{U_v})_i (\mathrm{P})}{\Delta \mathrm{U_i}} \quad . \tag{1.7}$$

Where ΔU_0 is a certain value for ΔU_i and we suddenly observing long fluctuation in the ratio n_i, this happens as the dimensions of ΔU_i approach those of a single pore. Finally as $\Delta U_i \to 0$ converging on the mathematical point P, n_i will become either one or zero depending on whether P inside pore or inside the solid matrix of the medium. [5]

(1.1.9) Permeability

Permeability is simply defined as the conductance of the porous medium via the Darcy's law as follows:

$$\frac{R}{A} = \left(\frac{K}{\mu}\right)\left(\frac{\Delta P}{L}\right) , \tag{1.8}$$

Equation (1.8) is the physical law, relating the volumetric flow rate R through a porous medium having a normal area A under the pressure gradient $\left(\frac{\Delta P}{L}\right)$, μ is the viscosity of the fluid, and K is the permeability of the porous medium. In other words, it is a measure of the resistance to fluid flow, and generally depends upon the pore size distribution, length, entrance, and exists of the pores, etc. The permeability of a porous medium is expressed in terms of "Darcy". A porous material is said to have permeability of one Darcy if a pressure difference of $1\ gm/cm.s^2$ at results in a flow of $1\ cm^3/s$ of a fluid having viscosity of $1\ gm/cm.s$ through a cube (of porous matrix) having sides $1\ cm$ in length.[6]

(1.2) The difference between Newtonian and non-Newtonian fluids

(1.2.1) Introduction

Many researches have been done in the field of fluid dynamic not only for its academic interest, but also for its industrial applications. The simple Newtonian fluid model has been considered as the standard fluid behavior for along time. Though most gases and low molecular weight substances to exhibit this kind of fluid behavior, in recent years, there has been an increasing recognition of anther kind of fluid called non-Newtonian fluid characteristics displayed by most materials encountered in every day life, both in nature (proteins, biological fluids, etc) and in technology (polymers and plastics, slurries, etc) are some examples of such fluids.

(1.2.2) Definition of Newtonian fluids

A fluid whose stress at each point is linearly proportional to its strain rate at that point is called Newtonian fluid, that is

$$\tau = \mu \, \dot{\gamma} \tag{1.9}$$

where τ is the shear stress, $\dot{\gamma}$ is the strain rate and μ is the viscosity coefficient. Therefore, the graph of this equation is a straight line of slope μ passing through origin. The single constant μ completely characterizes the laminar flow behavior of a Newtonian fluid at a fixed temperature and pressure.

(1.2.3) Non-Newtonian fluid behavior

The simplest possible deviation from the Newtonian fluid behavior occurs when the simple shear data $\tau - \dot{\gamma}$ does not pass through the origin and/ or does not result into a linear relationship between τ and $\dot{\gamma}$ Conversely, the apparent viscosity, defined as $\tau / \dot{\gamma}$, is not constant and is a function of τ or $\dot{\gamma}$. Indeed, under appropriate circumstances, the apparent viscosity of certain materials is not only a function of flow conditions (geometry, rate of shear, etc.), but it also depends on the kinematic history of the fluid element under consideration.

(1.2.4) Classification of non-Newtonian fluids [7-11]:

Non-Newtonian fluids may be classified into three general categories:-

1. Systems for which the value of $\dot{\gamma}$ at a point within the fluid is determined only by the current value of τ at that point; these substances are variously known as "purely viscous", "inelastic", "time-independent" or "generalized Newtonian fluids (GNF)".

2. Systems for which the relation between τ and $\dot{\gamma}$ shows further dependence on the duration of shearing and kinematic history; these are called time-dependent fluids.

3. Systems which exhibit a blend of viscous fluid behavior and of elastic solid-like behavior. For instance, this class of materials shows partial elastic recovery, recoil, creep, etc. Accordingly, these are called visco-elastic or elastico-viscous fluids.

As noted earlier, the aforementioned classification scheme is quite arbitrary, though convenient, because most real materials often display a combination of two or even all these types of features under appropriate circumstances. For instance, it is not uncommon for a polymer melt to show time-independent (shear-thinning) and visco-elastic behavior simultaneously and for a china clay suspension to exhibit a combination of time-independent (shear-thinning or shear-thickening) and time-dependent (thixotropic) features at certain concentrations and /or at appropriate shear rates. Generally, it is, however, possible to identify the dominant non- Newtonian aspect and to use it as basis for the subsequent process calculations. Each type of non-Newtonian fluid behavior is now dealt with in more detail.

(1.2.4.1) Time independent fluid behavior

As noted above, in simple unidirectional shear, this sub-set of fluids is characterized by the fact that the current value of the rate of shear at a point in the fluid is determined only by the corresponding current value of the shear stress and vice versa. Conversely, one can say that such fluids have no memory of their past history. Thus, their steady shear behavior may be described by a relation of the form,

$$\frac{du}{dy} = \dot{\gamma} = f\ (\tau) \tag{1.10}$$

Or, its inverse form,

$$\tau = f^{-1}(\dot{\gamma}) \tag{1.11}$$

Depending upon the form of equation (1.10) or (1.11), three possibilities exist:

1. Shear- thinning or pseudoplastic behavior

2. Visco-plastic behavior with or without shear-thinning behavior

3. Dilatant or shear- thickening behavior.

Figure 1.2 shows qualitatively the flow curves (also called rheograms) on linear coordinates for the above noted three categories of fluid behavior; the linear relation typical of Newtonian fluids is also included in figure 1.2.

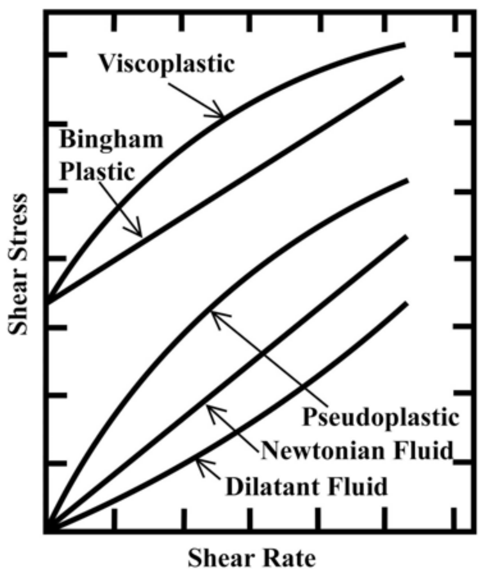

Fig. 1.2 Qualitative flow curves for different types of non- Newtonian fluids