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# استقرار الاتزان الكهروهيدروديناميكى لموائع متعددة الطبقات تنساب على مستوى مائل

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كجزء متمم للحصول على درجه الماجستير لا عداد المعلم فى العلوم
(رياضيات تطبيقية)

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# Electrohydrodynamic Stability of Multi-Layered Fluids Flowing Down an Inclined Plane

**A Thesis**

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(Applied Mathematic)

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# Chapter 1

# Introduction

## 1.1 Stability of liquid flow down an inclined plane

The stability of a liquid film flowing down an iclined plane was attacked by Kapitza [78], [79], among others. The first rigorous formulation of the problem, without taking surface tension into account, was given by Yih [22], who also gave the method of solution by expansion of it in a power series of $\alpha R$, $\alpha$ being the wave number and $R$ the Reynolds number. Yih's numerical calculation for the neutral stability curve was, however, quite inaccurate, and gave a neutral stability curve (for $c_i = 0$) which turns out to be more like a curve on which $c_i$ is nonzero but small. It also gave a value for the speed of propagation $c_r$ which turns out to be too large. In a paper based on Yih's formulation, but with surface tension taken into account, Benjamin [96] performed a new calculation, and obtained the condition for neutral stability analytically, instead of numerically. He used the method of power expansion in $y$, and his results, for small wave numbers only, are more in accord with the experimental results of Binnie [8]. This raised the question of whether the expansions used by Yih and Binjamin would give the same results, if the calculations are done correctly. Yih [26] showed that his method of expansion in $\alpha R$ or $\alpha$, when the calculations are carried out analytically, give the same results as Benjamin's for small $\alpha$, and further gave results for small Reynolds numbers and any wave number, and for large wave numbers and any finite Reynolds number. He also gave a physical interpretation of the rather puzzling situation of neutral stability for vertical flow with no surface tension at zero Reynolds number. The highly damped modes near $\alpha = 0$, $R = 0$, and for very large $\alpha$

are also shown by him to be vestiges of Tollmien-Schlichting waves. But the most important contribution remains Benjamin's. The following abridged account follows closely the paper of Yih [26].

The surface waves that can form at small Reynolds numbers, when the angle of inclination $\beta$ (Fig. (1.1)) is small, are quite different from the Tollmien-Schlichting waves, or shear waves. At low Reynolds numbers both kinds exist, except that the surface waves may be unstable whereas the shear waves are highly damped. At very small values of $\beta$ surface waves can form only at large values of the Reynolds number, and it is not yet clear whether surface waves are less stable than shear waves.

The flow considered is one half of plane Poiseuille flow, with a free surface replacing the center plane of the latter. With the presence of the free surface, gravity effect comes into play. Gravity has been found to be both stabilizing and destabilizing for $\beta < \pi/2$. It is destabilizing because, in the first place, the flow is caused by gravity, and, in the second place, the presence of the free surface enables its longitudinal component to play a destabilizing role. It is, for $\beta < \pi/2$, nevertheless stabilizing because its transverse component obviously stabilizes. On the other hand, surface tension is always stabilizing. The outcome of the conflict between the stabilizing and destabilizing effects determines the stability or instability of the flow.

The stability of two superposed layers of homogeneous liquids between parallel inclined planes has been studied by Graebel [104] for the special case of equal and opposite discharges and equal depth, and by Sangester (unpublished) for the general case. Further results on the stability of parallel flows of an incompressible fluid with variable density and viscosity have been given by Drazin [75]. post-instability mixing at the interface of two counter-flowing streams of different densities has been studied by Macagno and Rouse at the Iowa Institute of Hydraulic Research [38]. (Turbulent mixing at the interface of fresh water and salt water, due to a grid oscillating above it, has been studied by Rouse and Dodu [49]. But that is not a stability problem.)

The stability of the laminar flow of a liquid layer analyzed inexactly by Kapitza [78], [79] was first rigorously formulated by Yih [22], who solved the Orr-Sommerfeld equation by an expansion in powers of $\alpha R$. The resulting secular equation was solved by numerical computation, involving the solution of simultaneous nonlinear algebraic equations. Whereas the numerical computation produced the result that the flow down a vertical plane is unstable for Reynolds numbers larger than 1.5, thus establishing the instability of the flow at low Reynolds numbers, it was not accurate enough, and both the shape of the neutral stability curve and the values of the wave speed given are incorrect. In a paper based on Yih's formulation and on a variation of his method, Benjamin [96] performed a new calculation, with the important difference that his neutral stability curves were obtained analytieally, instead of numerically. His calculation established the result that free-surface fiow down a vertical plane is unstable for all finite Reynolds numbers, and gave values for the wave speed which are more in accord with experiments Binnie [8].

Yih's numerical computation and Benjamin's power expansion are both very laborious. It is desirable to have a simple method for the solution of problems of the same kind. A perturbation procedure based on Yih's expansion provides just such a method. The agreement of the results obtained by this new method with Benjamin's should dispel the feeling in the minds of some of the people working on free-surface instability that there is a fundamental difference between Yih's expansion and Benjamin's. But quite apart from this, the perturbation procedure provides a powerful method for solving stability problems involving free surfaces or interfaces, and is itself worth presenting.

The plane Poiseuille flow is known to be unstable only at rather high Reynolds numbers. Since free surface flow is one-half the plane Poiseuille flow, it is rather surprising that the free surface should make it unstable at very much lower Reynolds numbers. Should not there remain some features of the stability of the free-surface flow which are similar to those of the plane Poiseuille flow? Why should the features of the stability of the plane Poiseuille flow disappear so completely when a free surface is present? Clarification of this point leads not only to the understanding of the correct choice of mathematical approximations to be made in dealing with problems of free-surface instability, but also to a better understanding of the physies of the phenomenon.

Bebjamin's calculation is based on the assumption that $\alpha$ is small. For this reason Benjamin did not consider his calculation applieable to values of $\alpha$ which are not small. For the case of vertical flow with zero surface tension, he gave the dashed line $\alpha = 0.43$ (approximately) as the estimated neutral stability curve. This is incorrect, and has misled some people to obtain such a neutral-stability curve with a high-speed computer. Here Yih's method,[22] coupled with the new perturbation procedure, provides results at low Reynolds numbers for any value of $\alpha$, however large. These results, which cannot be obtained by Benjamin's power series expansion. They show that the entire axis $R = 0$ is part of a neutral stability curve for vertical film flows if surface tension is zero, and that there is no bifureation point enabling the curve to branch out. The greater versatility of the expansion in powers of $\alpha R$ is thus demonstrated.

The question at large values of the wavenumber $\alpha$ has so far not been touched. The pertinent result furnishes a new example of the dual role of viscosity, i.e., a new example of the destabilizing effect of viscosity.

The free-surface boundary condition involving shear will be formulated with variable surface tension taken into account, and the effect of this variability is briefly assessed.

Consider the flow of two immiscible, incompressible fluids in an inclined channel, separated by an interface with surface tension. The flow is driven by the component of gravity along the channel wall, and by an imposed pressure gradient. Since the fluids may have distinct physical properties, the interface is susceptible to the viscosity-stratification instability found by Yih [27] the instabilities associated with density stratification, shear-flow instabilities, as well as the interfacial instability found in single-phase falling films. It has been observed in core-annular flows that countercurrent flows are stable provided that the fluid in the thin annular region is less viscous than that in the interior,[18], [31], [36], [37], [46], [47], [58], [59], [63], [66], and [83] and is called the "thin-layer" effect by Hooper [9]. Where an adverse shear on the interface results from the viscosity difference, thereby stabilizing the single-phase falling-film instability.

The study of two-layer flows in a horizontal channel has been extensive. Yih [27] first considered the long-wave linear stability of the flow of two fluids of equal density, and found that an instability develops for any nonzero flow rate, when the fluids occupy equal volumes. Blennerhassett [77] examined the linear stability of Poiseuille flow of an air-water system, and found that a unit-order critical wave number marks the primary instability for sufficiently thick channels. Miles [57] and Smith and Davis [70] examined numerically the linear stability of a horizontal fluid layer under an imposed shear, and found that a unit-order wave-number instability devolps. Yiantsios and Higgins [88] extended Yih's work to arbitrary fluids and volume fractions, and found that hydrostatic effects can stabilize the viscosity-stratification instability for sufficiently small shears. Tilley, Davis and Bankoff [13] extended the above results to find for which mean interfacial heights and channel thicknesses the long-wave and the nonzero-wave-number instabilities are preferred. By doing so, they found situations in which the odd Orr-Sommerfeld shear mode in the water layer, stable in single-phase plane Poiseuille flow, can be destabilizing. This effect has not been reported in the literature. Goussis and Kelly [30] performed an energy analysis of two-layer Couette-Poiseuille flow in an inclined channel for cocurrent flows in the direction of gravity,and found that density stratification manifests its instability through perturbation shear stresses at the deformed interface, while viscosity stratification acts through perturbation velocities across the perturbed interface. Further studies have also been performed on the three-layer problem, as a prologue to the studies of core-annular flow [81], [103], and [110].

The interfacial instability of single-phase falling films has been studied since Benjamin [96] and Yih [22] analyzed its long-wave linear stability. This instability onsets at zero wave number, and from Floryan, Davis and Kelly [54] is the primary instability for sufficiently thin films and slightly inclined planes. The instability is marked by afinite bandwidth of modes becoming unstable for any nonzero parallel flow, in contrast to the viscosity-stratification instability described above. Smith [67] described the different possible mechanisms for a long-wave falling-film instability. One is the "velocity-induced" instability, driven by the velocity gradient normal to the flow of the basic state. The second is a "shear-induced" instability, which is driven by the gradient of the shear stress normal to the flow of the basic state. Tilley, Davis and Bankoff [13] completed Smith's description of these instabilities as a means of articulating the physics involved.

Yih [23] showed for the linear stability problem that three-dimensional disturbances of the two-layer flow in an inclined channel can be written in terms of a two-dimensional stability problem. Joseph and Renardy [32] discuss the distinction between Squire's transformation (converting a three-dimensional linear stability problem into a two-dimensional linear stability problem) and Squire's theorem [44] (the primary instability of a two-dimensional flow is to a disturbance in the plane of the flow). The essence of Squire's result for plane Poiseuille flow rests on the fact that the flow is unidirectional, and hence the Reynolds number is always a positive quantity. A similar result holds for two-layer flows in a horizontal channel, provided that density stratification is stabilizing (Hesla, Pranckh, and Preziosi [97]). The sign of the Reynolds numbers need not be considered because of the left-right symmetry of the horizontal channel: instabilities which occur in a right-moving flow are the same as those which occur in a left-moving flow.

However, the inclined channel does not possess this left-right symmetry, and hence one must consider both the sense and the magnitude of the Reynolds number of each fluid. This allows for the possibility that a two-dimensionally stable countercurrent flow may be unstable to some three-dimensional disturbance. Nonetheless, this new stability problem is encompassed by their study, although with new parameter values dictated by Squire's transformation.

One motivation for this study is the understanding of the phenomenon of flooding in an inclined channel. This phenomenon is found in countercurrent flows, and is characterized by the transition from a countercurrent flow to a cocurrent flow adverse to gravity when the adverse pressure gradient is increased in magnitude. During this transition, a variety of interfacial dynamics is observed ranging from possibly chaotic small-amplitude waves to large-amplitude waves that impede the flow of the second phase. Countercurrent flow returns only after the adverse pressure gradient is decreased below the flooding point, a process which is called flow reversal. Bankoff and Lee [87] reviewed the wide variety of theoretical and experimental attempts to explain the phenomenon, and argued that the analyses are inconclusive. Chang [45] used a phenomenological model to allow for the imposition of a turbulent shear stress on the interface by an otherwise passive phase, and derived a Kuramoto-Sivashinsky equation, valid for long waves, in a frame moving with the linear phase speed. Fowler and Lisseter [4] have used a phenomenological model to examine flooding in core-annular flow, and found a hysteresis loop between countercurrent and cocurrent states.Tilley, Davis, and Bankoff [14] derived a strongly nonlinear evolution equation, valid for long waves, and find a necessary condition for the trasition between smaller-amplitude traveling waves, and larger-amplitude traveling waves, which has been viewed as a precursor to the onset of flooding. That study described the parameter ranges for which such strongly nonlinear evolution equations are a valid mathematical representation of the physical interfacial motion (see Tilley, Davis, and Bankoff [13]).

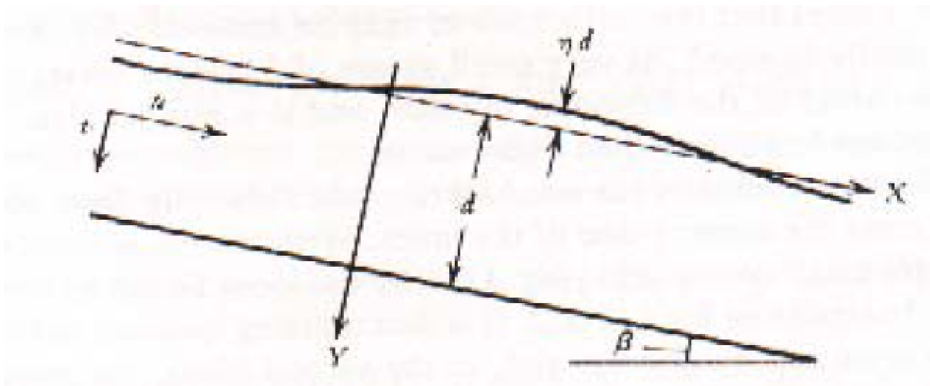


Figure (1.1)

(Definition sketch for free-surface flow down an inclined plane)

## 1.2 Basic Equations of motion and Boundary Conditions

The primary flow is assumed steady and parallel to the $X$-axis, with the velocity $\overline{u}$ varying only with $Y$ (Fig. (1.1)). The pressure gradient in the $X$-direction and the velocity component parallel to $Y$ are zero, so that the Navier-Stokes equations reduce to

$$\rho g \sin\beta + \mu \frac{d^2\overline{u}}{dY^2} = 0, \tag{1.1}$$

$$\frac{d\overline{p}}{dY} = \rho g \cos\beta, \tag{1.2}$$

in which $\rho$ is the density, assumed constant, $g$ the gravitational acceleration, $\mu$ the viscosity, $\overline{p}$ is the pressure of the primary flow, and $\beta$ is the angle of inclination of the plane boundary.

At $Y = d$ the fluid must adhere to the fixed boundary, and hence $\overline{u} = 0$, At $Y = 0$ the shear stress must vanish, and hence $d\overline{u}\ /dY = 0$. Integration of (1.1) with these boundary conditions yields

$$\overline{u} = \frac{g \sin\beta}{2\nu}(d^2 - Y^2)$$

or

$$U(y) = \frac{3}{2}(1 - y^2), \tag{1.3}$$

in which

$$U = \frac{\overline{u}}{\overline{u}_a}, \quad \overline{u}_a = \frac{gd^2 \sin\beta}{3y}, \quad y = \frac{Y}{d} \tag{1.4}$$

$\overline{u}_a$ is theaverage velocity of the primary flow.

If the Reynolds number and Froude number are defined to be

$$R = \frac{\overline{u}_a d}{\nu} \quad \text{and} \quad F = \frac{\overline{u}_a}{\sqrt{gd}}, \tag{1.5}$$

the second equation in (1.4) can be written as

$$3F^2 = R \sin\beta. \tag{1.6}$$

With the origin of the Cartesian coordinates (shown in Fig. 1.1) at the free surface, and with $u$ and $v$ denoting the velocity components in the directions of $X$ and $Y$, respectively, the equation of continuity is

$$\frac{\partial u}{\partial X} + \frac{\partial v}{\partial Y} = 0,$$

and the Navier-Stokes equations are

$$\begin{aligned}
\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial X} + v\frac{\partial u}{\partial Y} &= -\frac{1}{\rho}\frac{\partial p}{\partial X} + g\sin\beta + \nu\nabla^2 u, \\
\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial X} + v\frac{\partial v}{\partial Y} &= -\frac{1}{\rho}\frac{\partial p}{\partial Y} + g\cos\beta + \nu\nabla^2 v,
\end{aligned}$$

in which $t$ is the time, $p$ the pressure, and $\nabla^2$ the Laplacian operator. With the substitutions

$$(u_1, \upsilon_1) = \frac{(u, \upsilon)}{\rho \overline{u}_a}, \quad (\varkappa, y) = \frac{(X, Y)}{d}, \quad p_1 = \frac{p}{\overline{u}_a^2}, \quad \tau = \frac{t\overline{u}_a}{d}$$

the equations of motion and of continuity assume the following dimensionless forms:

$$\frac{\partial u_1}{\partial \tau} + u_1 \frac{\partial u_1}{\partial \varkappa} + \upsilon_1 \frac{\partial u_1}{\partial y} = -\frac{\partial p_1}{\partial \varkappa} + \frac{\sin\beta}{F^2} + \frac{1}{R}\nabla^2 u_1 \tag{1.7}$$

$$\frac{\partial \upsilon_1}{\partial \tau} + u_1 \frac{\partial \upsilon_1}{\partial \varkappa} + \upsilon_1 \frac{\partial \upsilon_1}{\partial y} = -\frac{\partial p_1}{\partial y} + \frac{\cos\beta}{F^2} + \frac{1}{R}\nabla^2 \upsilon_1 \tag{1.8}$$

$$\frac{\partial u_1}{\partial \varkappa} + \frac{\partial \upsilon_1}{\partial y} = 0 \tag{1.9}$$

Let

$$u_1 = U + u', \quad \upsilon_1 = \upsilon', \quad p_1 = P + p', \tag{1.10}$$

in which $U$ and $P$ are the (dimensionless) velocity and pressure of the primary flow, and the accented quantities are perturbation quantities. Substitution of (1.10) into (1.7), (1.8), and (1.9) yields, with subscripts denoting partial differentiation,

$$u'_\tau + U u'_\varkappa + U_y \upsilon' = -p'_\varkappa + \frac{1}{R}\nabla^2 u', \tag{1.11}$$

$$\upsilon'_\tau + U \upsilon'_\varkappa = -p'_y + \frac{1}{R}\nabla^2 \upsilon', \tag{1.12}$$

$$u'_\varkappa + \upsilon'_y = 0, \tag{1.13}$$

if quadratic terms of the perturbation quantities are neglected. In obtaining (1.11) and (1.12), the fact that $U$ and $P$ satisfy the Navier-Stokes equations has been utilized.

As a consequence of (1.13),

$$u' = \psi_y, \quad \upsilon' = -\psi_\varkappa$$

in which $\psi$ is the dimensionless stream function for the velocity perturbation. Equations (1.11)