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LATERAL TORSIONAL BUCKLING OF TAPERED I-BEAMS

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STATEMENT

This dissertation is submitted to Ain Shams University in partial fulfillment for the requirements of the degree of Doctor of Philosophy in Structural Engineering.

The work included in this thesis was carried out by the author in the Structural Engineering Department, Ain Shams University.

No part of this thesis has been submitted for a degree or a qualification at any other University or Institution.

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بسم الله الرحمن الرحيم

"قـالـوا سبحانك لا علـم لنـا إلا مـا علمتـنـا إنـك أنـت العلـيم الحكـيم"

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Abstract of the Ph. D. Thesis submitted by: Eng. Esam Abdel- Kader Naguib Wally.

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ABSTRACT

Modern techniques of fabricating plated steel members have resulted in an increased use of tapered elements in steel structures. This is reflected on the material cost by reducing the member cross section in the low moment regions. Tapered beams are fabricated from an arbitrary numbers of flat plates joined by welding along their edges, so that the cross- section consists of a set of connected thin rectangles. When a tapered member does not have an adequate lateral support, its strength is governed by its resistance to torsional buckling modes, one of them is lateral torsional buckling

The present research aims to evaluate the lateral torsional buckling of tapered I- beams. This is achieved by adopting two different techniques; variational approach and finite element method. Variational approach to derive the Euler- Lagrange equations for the displacement components, the equilibrium equations defining instability phenomena and the corresponding limit conditions are obtained for thin, open cross-section, continually tapered beams. The variability of the cross-section along the span introduces an additional term for the expression of the tangential torsions due to variation of the shear centre position. A theoretical analysis was carried out on the basis of the positive definiteness of the second variation of the total potential energy which represents the stability criterion of the beams. Closed form solutions for different cases of tapered beams are presented.

As an alternative analysis method; finite element (F.E) model is presented for tapered I- beams. Material and geometric nonlinearities are incorporated in the F.E. model. Overall geometric imperfection, which causes lateral torsional buckling to I- beams, is incorporated in the model, as well. Applications are carried out on variable cross-sections of tapered I- beams to calculate critical lateral torsional buckling stresses.

A parametric study is performed on simply supported tapered I-beams with doubly symmetric or mono-symmetric cross sections. Several parameters such as tapering ratios (web tapering, flange tapering), spans, flanges thicknesses, and different types of loading are investigated and their effect on lateral torsional buckling is analyzed.

A set of imperical equations are proposed to predict the lateral torsional buckling load for different types of tapered I- beams.

Finally conclusions and recommendations for future work are presented.

Key words: tapering ratio, symmetric, mono-symmetric, lateral torsional buckling, I-beams.

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