GALERKIN/LEAST-SQUARES FEM ON A GPU ARCHITECTURE FOR VISCOELASTIC FLUIDS

By

Mahmoud Mohamed Mohamed Ahmed Ayyad

A Thesis Submitted to the
Faculty of Engineeringat Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
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in
Engineering Mathematics

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Key Words:

Finite Element Method, Graphics Processing Units, Galerkin/least-squares-method, Viscoelastic Fluids

Summary:
The Galerkin/Least-Squares Finite Element Method (FEM) is used to simulate the flow of blood, modeled as viscoelastic fluid, in abdominal aorta with two aneurysms. Discrete Elastic-Viscous Stress-Splitting (DEVSS) is used to overcome the instability that arises from considering the blood as a viscoelastic fluid. The solution is accelerated by implementing the FEM on graphics processing unit (GPU). The problem is implemented on many-core CPU and multi-core GPU architectures. Numerical experimental results find that the proposed algorithm on the GPU architecture shows a significant speed-up over the CPU architecture implementations.

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List of Symbols and Abbreviations

ρ	Density
t	time
$\mathbf{V}$	Velocity vector
∇	Vector differential operator
τ	Extra stress tensor
P	Pressure
μ_s	Shear-rate viscosity
D	Rate of strain tensor
λ	Relaxation time
$\tau_{\mathbf{s}}$	Newtonian contribution on extra stress tensor
$\tau_{\mathbf{p}}$	Polymeric contribution on stress tensor
L	Characteristic length
u_∞	Reference velocity
p_∞	Free stream pressure
S	Polymeric axial stress
Q	Polymeric shear stress
T	Polymeric normal stress
Re	Reynolds Number
μ	Total viscosity
μ_p	Polymer shear-rate viscosity
β	Viscosity ratio
We	Weissenberg number
γ	Penalty coefficient
δ	Artificial coefficient
τ	Artificial time
ϵ	Pressure dissipation parameter

$\mathbf{B}$	Artificial stress tensor
u	Velocity component in x-direction
v	Velocity component in y-direction
N_i	Shape function
W_i	Weight function
$\hat{u}$	Average velocity component in x-direction at the centre of the element
$\hat{v}$	Average velocity component in y-direction at the centre of the element
ξ	Horizontal axis in the element local coordinates
η	Vertical axis in the element local coordinates
$\triangle t$	Time step
$\|\|\mathbf{u}^{\mathbf{e}}\|\|$	Vertical axis in the element local coordinates
α	Stabilization parameter in GLS weight function
$\mathbf{K}_{ie}$	ith local matrix
$\mathbf{K}_i$	ith global matrix
Λ	Aspect ratio
D	Artery diameter
D_1	First aneurysm diameter
D_2	Second aneurysm diameter
L_1	First aneurysm length
L_2	Second aneurysm length
L_T	Total aneurysm length
D_2	Second aneurysm diameter
N_e	Total number of elements
N_n	Total number of nodes
n_x	Number of nodes in x-direction
n_y	Number of nodes in y-direction
GPU	Graphics Processing Unit
CPU	Centeral Processing Unit