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شبكة المعلومات الجامعية التوثيق الالكتروني والميكرو فيلم



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جامعة عين شمس

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GROUP THEORETIC APPROACH TO LINEAR AND NONLINEAR PHYSICAL PROBLEMS

Thesis

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Introduction

Introduction

Group-theoretic methods are powerful, versatile and fundamental to the development of systematic procedures that lead to invariant solutions of differential equations. The group-theoretic methods are applicable to both linear and nonlinear differential equations since they are not based on linear operators, superposition or other requirements of the linear solution techniques.

A systematic investigation of continuous transformation group was carried by Lie [1-3]. His original goal was the creation of a theory of integration for ordinary differential equations analogous to the Abelian theory for the solution of algebraic equations. He investigated the concept of the invariance groups admitted by a given system of differential equations. These groups have important real world applications. A number of books on the application of the continuous groups of transformations relating to differential equations have been written from a mathematical standpoint, for example Bluman and Cole [4], Ovisannkov [5], Hill [6] Olver [7], Bluman and Kumei [8], Hans [9] and Ibragimov [10]. In addition, the works of Hansen [11], Ames [12,13] and Dresner [14], Na and Hensen [15] present quite extensively the general theories involved in the similarity solution of differential equations as applied to engineering and physics problems.

In this thesis, we applied a one-parameter Lie group method not only to find invariants of partial differential equations but also to predict the existence of invariants. Using this method, one can try to obtain invariant, and partially invariant solution to form the group in order to transform a given partial differential equation to a less complicated or ordinary differential equation [7,8,13].

Similarity transformations essentially reduce the number of independent variables in partial differential equations by one. Hence, for partial differential equations which have two independent variables, the similarity transformations transform a partial differential equation into an ordinary differential equation and make the determination of a class of solutions possible for the given partial differential equation depending on arbitrary constants. Similarity transformations have been also used to convert moving boundary conditions to constant boundary conditions. There has been considerable interest in symmetry reductions of partial differential equations, mainly because the procedure reduces the number of independent variables, and, therefore, assists in the determination of exact solutions.

The classical method [5,6,13,15] for obtaining the similarity reductions is using the symmetry properties of the partial differential equations. It is possible to obtain the necessary and sufficient conditions for invariance of the partial differential equations with respect to the one-parameter group of transformations. Setting all the coefficients of like derivatives equal zero, one obtains a system of determining equations for the group elements. The group of points symmetries which leaves the partial differential equations invariant may be determined by means of its generators. These infinitesimal generators must form a Lie algebra determined by the structure constants. This means that the set of generators chosen must be closed under the commutative operator. Having defined the generators of a symmetry group, the invariance reduction allows determination of the similarity variable and the form of similarity solution, using the general integral of the characteristic system. The symmetry transformation group can be generated from the corresponding Lie algebra. In fact to find the group we only have to find its Lie algebra. We will seek the Lie algebra and determine its corresponding transformation group.