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The Department of Mathematics, Faculty of Education,
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By

Naglaa Fawzi Abd Allah

Supervised by

Prof.Dr.Mohamed Fahmy El-Sayed
Professor of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

Prof. Dr. Hassan Nasr Ahmed Ismail
Emeritus Professor of Pure Mathematics
Department of Mathematics
Faculty of Engineering
Banha University

Dr. Emad Hassan Aly
Assistant Professor of Applied Mathematics
Department of Mathematics
Faculty of Education
Ain Shams University

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Candidate: Naglaa Fawzi Abd Allah

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Supervisors:

No.	Name	Profession	Signature
1.	**Prof. Dr. Mohmed Fahmy El-Sayed**	Professor of Applied Mathematics, Mathematics Department, Faculty of Education, Ain Shams University.	
2.	**Prof. Dr.Hassan Nasr Ahmed Ismail**	Emeritus Professor of Pure Mathematics, Mathematics Department, Faculty of Engineering, Banha University.	
3.	**Dr. Emad Hassan Aly**	Assistant Professor of Applied Mathematics, Mathematics Department, Faculty of Education, Ain Shams University.	

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Summary

This thesis contains numerical solutions for systems of nonlinear equations governing fluid flow and heat transfer of some non-Newtonian fluids through different geometric shapes. Also presented a study to the error analysis in numerical methods by comparing it with exact solution and previously published work. It should be noted that the solution of the current results is obtained by designing Matlab programme and then the present graphics are drawn by designing Excel and Matlab programmes. This thesis consists of four chapters, which are followed by lists of references.

Chapter(1)

The introductory chapter is considered as a background for the material included in the thesis. The purpose of this chapter is to present a short introduction on numerical analysis and fluid mechanics, a brief survey of famous numerical methods which using to solve fluid mechanics problems, fluid properties and the basic flow equations. Moreover, it contains a short survey of some needed concepts of the material used in this thesis.

Chapter(2)

The purpose of this chapter is to study the effect of Casson viscosity on steady MHD flow and heat transfer between two parallel plates in the presence of dissipations. the governing equations are transformed into ordinary differential equations by applying the dimensionless quantities. In addition, the resulting equations solved numerically by using the finite difference method (FDM). Moreover, numerical results are presented for the distribution of velocity and temperature profiles for various parametric conditions. The effects of varying pres-

sure parameter α, the Hartman number H_a and the yield stress parameter τ_D are determined. Furthermore, at the end of this chapter the conclusions are summarized. Some results of this chapter is accepted **(Asian Journal of Mathematics and Computer Science)**.

Chapter(3)

The aim of this chapter is to study the effect of radiation, heat generation and dissipations on heat transfer of stagnation point MHD flow of micropolar fluid over a stretching sheet. Using suitable similarity transformations, the governing partial differential equations are transformed into ordinary differential equations and then solved numerically by applying (FDM). The solutions are found to be governed by six parameters, the stretching parameter C, the material parameter K, the thermal radiation parameter R_d, the Prandtl number P_r, the heat generation/absorption parameter B and Eckert number E_c. Numerical results are presented the distribution of velocity and temperature profiles. Furthermore, comparisons of the present results with previously published work respect to skin friction show that the present results have high accuracy and are found to be in a good agreement. At the end of this chapter, the conclusions are summarized. The work in this chapter is submitted to (**International Journal of advances in Applied Mathematics and Mechanics**).

Chapter(4)

The main goal of this chapter is to study the effect of Hematocrit viscosity on the blood (non-newtonian fluid) flow and temperature through a rectangular artery of large aspect ratio in the presence viscous of dissipation. The non-dimensional quantities are applied to transform the governing equations into ordinary differential equations. Numerical solutions of the governing (momentum and energy) equations are obtained taking suction and injection into consideration. The nonlinear system of equations linearized using finite difference method to obtain the velocity and temperature distributions. Numerical and graphical results for the velocity and temperature profiles are presented and discussed for various parameters. Figures and Tables illustrate the effects of dimensionless non-Newtonian viscosity β, suction parameter S and Brinkman number B_r on the nondimensional velocity and tem-

perature. Furthermore, comparisons of the present results with exact solution at a specific case have high accuracy and are found to be in a good agreement. Some results of this chapter is accepted for **(Journal of Natural Sciences and Mathematics)**.

Chapter 1

Introduction

Biomathematics is the use of mathematical models to help understand phenomena in biology. Scientists routinely use advanced mathematics to describe how the heart works, how blood flows, how nerve impulses transmit, how tumors grow and how entire organisms grow. Where Mathematical models are important tools in basic scientific research in many areas of biology, including physiology, ecology, evolution, toxicology, immunology, natural resource management, and conservation biology. Thus, while mathematical biology may sound like a narrow discipline, in fact it encompasses all of biology and virtually all of the mathematical sciences, including statistics, operations research, and scientific computing. Today theoretical/mathematical biology is booming; currently it seems to offer lots of promising perspectives and possibilities for mathematicians and theoretically interested biologists [31].

The introductory chapter is considered as a background for the material included in the thesis. The purpose of this chapter is to present a short introduction on numerical analysis and fluid mechanics and a brief survey of fluid properties and the basic flow equations. Moreover, it contains a short survey of some needed concepts of the material used in this thesis with a great of many enrichment details.

Part one: Numerical Analysis

Numerical analysis is the area of mathematics and computer science that creates analyzes and implements algorithms for solving numerically the problems of continuous mathematics. Such problems originate generally from real-world applications of algebra, geometry and calculus. These problems involve variables which vary continuously and occur throughout the natural sciences, social sciences, engineering, medicine and business. During the past half-century, the growth in power and availability of digital computers has led to an increasing use of realistic mathematical models in science and engineering. The formal academic area of numerical analysis varies from quite theoretical mathematical studies to computer science issues [3]. With the growth in importance of using computers to carry out numerical procedures in solving mathematical models of the world an area known as scientific computing or computational science. This area looks at the use of numerical analysis from computer science perspective [32]. It is concerned with using the most powerful tools of numerical analysis, computer graphics, symbolic mathematical computations simulation and graphical user interfaces to make it easier for user to set up solve and interpret complicated mathematical models of the real world.

1.0.1 Historical background

Numerical algorithms are almost as old as human civilization. The Rhind Papyrus (1650 BC) of ancient Egypt describes root finding method for solving simple equation. Archimedes (287-212 BC) created much new mathematics including the method of exhaustion for calculating lengths, areas and volumes of geometric figures. It was an important precursor to the development of the calculus by Isaac Newton and Gottfried Leibnitz. A major impetus to developing numerical procedures was the invention of the calculus by Newton and Leibnitz as this led to accurate mathematical models for physical reality, first in the physical sciences and eventually in the other sciences, engineering, medicine and business. These mathematical models cannot usually be solved explicitly and numerical methods to obtain approximate solutions are needed. Another important aspect of the development of numerical methods was the creation of logarithms by Napier (1614) and others giving much simpler manner of carrying out the arithmetic operations of multiplication, division and exponentiation [25]. Newton created a number of numerical methods for solving variety of problems and his name is attached today to generalizations of his original ideas. For example his work on root finding and polynomial interpolation. Following Newton, many of the giants of mathematics of the 18^{th} and 19^{th} centuries made major contributions to the numerical solution of mathematical problems. Foremost among these are Leon-hard Euler (1707-1783), Joseph-Louis Lagrange (1736-1813) and Karl Friedrich Gauss (1777-1855). Up to the late 1800, it appears that most mathematicians were quite broad in their interests and many of them were interested in and contributed to numerical analysis [25].

1.0.2 Numerical linear and nonlinear algebra

This refers to problems involving the solution of systems of linear and nonlinear equations with a very large number of variables. Many problems in applied mathematics involve solving systems of linear equations. Linear systems are usually written using matrix-vector notation by form $Ax = b$, where A is the matrix of coefficients for the system, x the column vector of the unknown variables $x_1, x_2, ..., x_n$ and b is a given column vector. Solving linear systems with up to $n = 1000$ variables is now considered relatively straightforward in most cases. For small to moderate sized linear systems say ($n \leqslant 1000$), the favorite numerical method is Gaussian elimination and its variants. This is simply a precisely stated algorithmic variant of the method of elimination of variables that students first encounter in elementary algebra. For larger linear systems, there are a variety of approaches depending on the structure of the coefficient matrix A. Direct methods lead to a theoretically exact solution x in a finite number of steps with Gaussian elimination the best known example. In practice, there are errors in the computed value of x due to rounding errors in the computation arising from the finite length of numbers in standard computer arithmetic. Iterative methods are approximate methods which create a sequence of approximating solutions of increasing accuracy [26].

Nonlinear problems are often treated numerically by reducing them to sequence of linear problems. As a simple but important example consider the problem of solving nonlinear equation $f(x) = 0$. Approximate the graph of $y = f(x)$ by the tangent line at a point $x(0)$ near the desired root and use the root of the tangent line to approximate the root of the original nonlinear function $f(x)$. This leads to Newton's method for root finding:

$$x^{(k+1)} = x^k - \frac{f(x^{(k)})}{f'(x^{(k)})} \qquad k = 0, 1, 2, ... \tag{1.1}$$

This generalizes to handling systems of nonlinear equations. Let $f_n(x) = 0$ denote a system of n nonlinear equations in n unknowns $x_1, ..., x_n$. Newton's method for solving this system is given by:

$$x^{(k+1)} = x^k + \delta^k \tag{1.2}$$

$$f'(x^{(k)}\delta^k = -x^{(k)}, \qquad k = 0, 1, 2, ... \tag{1.3}$$

In this $f'(x)$ is the Jacobian matrix of $f(x)$ and the second equation is a linear system of order n. There are numerous other approaches to solving nonlinear systems based on using some type of approximation using linear functions [60].

1.1 Modern applications and computer software of numerical analysis

Numerical analysis and mathematical modelling have become essential in many areas of modern life. Sophisticated numerical analysis software is being embedded in popular software packages, e.g. spreadsheet programs, allowing many people to perform modelling even when they are unaware of the mathematics involved in the process. This requires creating reliable, efficient and accurate numerical analysis software. It requires designing problem solving environments (PSE) in which it is relatively easy to model a given situation. The (PSE) for a given problem area is usually based on excellent theoretical mathematical models made available to the user through a convenient graphical user interface. Such software tools are well-advanced in some areas, e.g.

computer aided design of structures, while other areas are still grappling with the more basic problems of creating accurate mathematical models and accompanying tools for their solution, e.g. atmospheric modelling.

1.1.1 Some application areas of numerical analysis

Computer aided design (CAD) and computer aided manufacturing (CAM) are important areas within engineering and some quite sophisticated (PSEs) have been developed for CAD/CAM. A wide variety of numerical analysis is involved in the mathematical models that must be solved. The models are based on the basic Newtonian laws of mechanics. An important (CAD) topic is that of modelling the dynamics of moving mechanical systems. The mathematical model involves systems of both ordinary differential equations and algebraic equations. The numerical analysis of these mixed systems called differential algebraic systems is quite difficult but important to being able to model moving mechanical systems. Building simulators for cars, planes and other vehicles requires solving differential-algebraic systems in real-time [11].

1.1.2 Computer software of numerical analysis

Software to implement common numerical analysis procedures is very important. If it is to be shared by many users it needs to be reliable, accurate and efficient. Moreover, it needs to be written so as to be easily portable between different computers. The most popular programming language for implementing numerical analysis methods continues to be Fortran and it too continues to be updated to meet changing needs with Fortran (95) being the most recent standard. Other languages are also important with the most important ones being C, C++ and Java. Another approach to supplying numerical analysis programs and programming tools has been to create higher level problem solving environments that contain numerical programming and graphical tools including some quite sophisticated numerical analysis tools to handle many basic problems. Best known of these is MATLAB. A commercial package that has arguably become the most popular way to do numerical computing. For analytical mathematics computing there are two popular commercial packages: Maple and Mathematica.

1.2 Ordinary differential equations

1.2.1 Initial-value problems

Consider the simplest case of a first-order ordinary differential equation having the general form

$$\frac{dx}{dt} = f(x,t) \tag{1.4}$$

where f is an analytic function. At a starting point $t = t_0$ the function x has a given value x_0 and it is desired to find $x(t)$ for $t > t_0$ that satisfies both (1.4) and the prescribed initial condition. Such a problem is called an initial..value problem (IVP) [9].
To solve this problem numerically the axis of the independent variable is usually divided into evenly spaced small intervals of width h whose end points are situated at

$$t_i = t_0 + ih, \qquad\qquad i = 0, 1, 2, ... \tag{1.5}$$

The solution evaluated at the point t_i is denoted by x_i. Thus, by using a numerical method the continuous function $x(t)$ is approximated by a set of discrete values $x_i,\ \ i = 0, 1, 2, ...$ Since h is small and f is an analytic function, then the solution at any point can be obtained by means of a Taylor's series expansion about the previous point [9]:

$$\begin{aligned} x_{i+1} \equiv x(t_{i+1}) = x(t_i + h) = x_i + h\left(\frac{dx_i}{dt}\right) + \frac{h^2}{2!}\left(\frac{d^2x_i}{dt^2}\right) \\ + \frac{h^3}{3!}\left(\frac{d^3x_i}{dt^3}\right) + ... = x_i + hf_i + \frac{h^2}{2!}f_i' + \frac{h^3}{3!}f_i'' + ... \end{aligned} \tag{1.6}$$

where f_i^n denotes $\frac{d^nf}{dt^n}$ evaluated at (x_i, t_i). f is generally a function of both x and t, so that the first-order derivative is obtained according to the formula

$$\frac{df}{dt} = \frac{df}{dt} + \frac{dx}{dt}\frac{\partial f}{\partial x}. \tag{1.7}$$