Faculty of Education

Department of Mathematics

Exact and Computational Methods for Solving some Integral and Differential Equations

A Thesis

Submitted in Partial Fulfillment of the Requirements of the Master Degree in Teacher Preparation in Science

(Pure Mathematics)

Submitted to

Department of Mathematics, Faculty of Education, Ain Shams University

By

Yara Mostafa Sayed Mohamed

Supervised by

Prof. Dr. Ahmed Younis Ghaly
Professor of applied Mathematics,
Department of Mathematics,
Faculty of Education,
Ain shams University.

Prof. Dr. Hassan Ahmed Zedan
Professor of Pure Mathematics,
Department of Mathematics,
Faculty of Science,
Kafr El-Sheikh University.

Dr. Seham Shapll Tantawy
Lecturer of Pure Mathematics,
Department of Mathematics,
Faculty of Education,
Ain shams University.

(2015)

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Faculty of Education,
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بِسْمِ اللهِ الرَّحْمنِ الرَّحِيمِ

(قَالُواْ سُبْحَانَكَ لاَ عِلْمَ لَنَا إِلاَّ مَا عَلَّمْتَنَا إِنَّكَ أَنتَ الْعَلِيمُ الْحَكِيمُ)

سورة البقرة - الآية (32)

الإهـــــداء

إن الحمد لله نحمده ونستعينه ونستغفره ونستهديه، ونصلي ونسلم على المبعوث رحمة للعالمين سيدنا محمد بن عبد الله عليه أفضل الصلاة والسلام. تعتبر الرسالة الإلهية لرسول الله محمد (صلي الله عليه وسلم) تحريرًا للعقول من الخرافات ودعوة للتأمل واكتشاف أسرار الكون " سَنُرِيهِمْ آيَاتِنَا فِي الآفَاقِ وَفِي أَنْفُسِهِمْ حَتَّىٰ يَتَبَيَّنَ لَهُمْ أَنَّهُ الْحَقُّ أَوَلَمْ يَكْفِ بِرَبِّكَ أَنَّهُ عَلَىٰ كُلِّ شَيْءٍ شَهِيد". [فصلت:53] وهناك العديد من النصوص التي تحث علي العلم وطلبه، لذا فإني أسأل الله أن يعلمنا ما ينفعنا وأن ينفعنا بما علمنا، وأن ينفع بهذا العمل.

يقول العالم (الفيلسوف) جاليليو:

(إن الرياضيات هي اللغة التي أذا فهمناها لأصبح في استطاعتنا فهم أسرار الكون).

وبهذه المناسبة يسعدني أن أهدي هذه الرساله التي أسأل الله أن يتقبلها وينفع بها إلى:

- مدرستي الأولى التي علمتني العطاء بلا مقابل، وغمرتني بالحنان والحب الخالصين، وتحملت صعاب الحياة عن طيب نفس، بارك الله في عمرك يا أمي وألبسك ثياب الصحة والعافية، وأحسن خاتمتك، ورزقك الفردوس الأعلى.
- من علمني الصمود مهما تبدلت الظروف إلى أبي - رحمه الله - الذي لولا تشجيعه وتحفيزه ما اكتمل هذا العمل جعله الله في ميزان حسناته ورزقه الفردوس الأعلى.
- من شملوني بالعطف، وأمدوني بالعون، وحفزوني للتقدم، وحبهم يجري في عروقي، إخوتي وأخواتي رعاهم الله.
- أســـــاتذتي الكرام، نجوم الهدى في ليل الظلام، من كلّت أناملهم ليمهدوا لي الطريق إلى الأمام، فلهم مني جـزيل الشكر والتقدير والاحترام.
- زملائـــي الأوفياء، من تجسدت فيهم معاني الحب والوفاء والصدق والعطاء.
- وأخيرًا أدعو الله- سبحانه وتعالى- أن يتقبل هذا الجهد وأن يجعله زخرًا لي يوم القيامة وأن ينفع به المسلمين.

يارة،،،
2015

شكر وتقدير

بداية أشكر من تفضل وتكرم، وأعطى وأنعم، ووفق ويسر، خالقي ورازقي وولي نعمتي ربي ورب كل شيء.

فالحمد لله الذي بنعمته تتم الصالحات، فلقد وفقني الله سبحانه بفضله وجوده ومنه وكرمه في إنجاز هذه الرساله. فالحمد لله على الدوام وله الشكر على التمام، فهو: "الرَّحْمَنُ .عَلَّمَ القُرْآنَ .خَلَقَ الإِنسَانَ .عَلَّمَهُ البَيَانَ " الرحمن(1-4).

والصلاة والسلام على سيد الأنام، وحبيب الرحمن سيدنا محمد - صلى الله عليه وسلم- من حثنا على طلب العلم، فهو معلم البشرية فصلاة وسلاما عليه ننال بهما في الدنيا عزة وكرامة، وفي الآخرة صحبة وشفاعة.

كما لا يسعني إلا أن أتقدم بعميق الشكر والإمتنان لهيئة الإشراف وهم:

الأستاذ الدكتور/ أحمد يونس غالي. أستاذ الرياضيات التطبيقية بكلية التربية - جامعة عين شمس لدعمه المستمر وتشجيعه المتواصل والذي علمني الكثير ليس فقط في الجانب العلمي ولكن أيضًا في جوانب الحياه المختلفة.

الأستاذ الدكتور/ حسن أحمد زيدان. أستاذ الرياضيات البحتة بكلية العلوم – جامعة كفر الشيخ الذي أحاطني بالرعاية والإهتمام من خلال اقتراحه لموضوعات البحث وإرشاده المستمر وصبره وعطائه اللامحدود وتوجيهه القيم، كما زودنى بحكمته والمعلومات العديدة من خلال العديد من المناقشات.

الدكتور/سهام شبل طنطاوى. مدرس الرياضيات البحتة بكلية التربية - جامعة عين شمس لدعمها المستمر من خلال تعليقاتها التى لا تقدر بثمن والنقاط المضيئة التي كانت أعظم مساعد لي أثناء إعداد الأطروحة. ولقد أعطتني الكثير من وقتها الثمين، كما زودتنى بالمعلومات العديدة من خلال العديد من المناقشات.

كما أتقدم بجزيل الشكر إلى قسم الرياضيات- كلية التربية - جامعة عين شمس ممثلا برئيس القسم **الأستاذ الدكتور/ حسن عجوة** الذي كان لنا خير مشجع، وإلى أساتذتي الكرام لما أحاطوني به من رعاية وإهتمام خلال سنوات الدراسة فلهم جزيل الشكر.

كما أتقدم بالشكر الجزيل إلى أمي وإخوتي وأخواتى الذين صبروا وتحملوا الكثير من أجلي فأنا مدينه لهم بكل شئ.

وأخيرا وليس آخرا، أتقدم بالشكر الجزيل إلى أبى-رحمه الله- لكل الحب والدعم والتشجيع الذى أظهره لى دائمًا.

يارة مصطفى

Abstract

This thesis consists of five chapters distributed as follows:

Chapter I

This chapter is an introduction to the basic concepts of integral equations. It includes the classifications of integral and integro-differential equations. It also includes an introduction to the basic definitions and the necessary properties for fractional calculus such as the definitions of Riemann-Liouville, Caputo and discusses some necessary mathematical definitions that will arise in the study of these concepts.

An introduction to the various methods used in this thesis to obtain the exact solutions and the numerical solutions. It also includes an introduction to some theorems and basic concepts of measure theory which will be used in this thesis.

Chapter II

In this chapter, we improve and extend variational iteration method (VIM) and Chebyshev spectral method to find the exact solutions and the approximate solutions for fractional differential equations, fractional integro-differential equations, nonlinear systems of fractional integro-differential equations and generalized Abel's integral equations of the second kind. Moreover, we aim to study the convergence of the VIM for fractional differential equations, fractional integro-differential equations, nonlinear systems of fractional integro-differential equations and generalized Abel's integral equations of the second kind and to address the sufficient condition for convergence. The results obtained by variational iteration method and Chebyshev spectral method in this chapter are compared with the exact solutions and with the results obtained by some other authors, this comparison shows that we obtained better results and more accurate.

Chapter III

In this chapter, we apply the differential transform method (DTM) and homotopy perturbation method (HPM) to solve fifth-order boundary value problem, system of second-order boundary value problem, system of Volterra integral equations, systems of linear and nonlinear integro-differential , Cauchy problem, boundary value problem of fractional order, fractional integro-differential equations and nonlinear systems of fractional integro-differential equations.

In addition, we extend the modified Laplace decomposition method (mLDM) and the modified Laplace decomposition method with the Padé approximant (mLD-PA) to solve boundary value problem of fractional order and systems of linear and nonlinear fractional integro-differential equations. The results obtained by differential transform method, fractional differential transform method, homotopy perturbation method,

modified homotopy perturbation method, modified Laplace decomposition method and modified Laplace decomposition method with the Padé approximant in this chapter are compared with the exact solutions and with the results obtained by some other authors, this comparison shows that we obtained better results and more accurate.

Chapter IV

In this chapter, we extend and generalize the Haar wavelet method (HWM) and Legendre wavelets method (LWM) to solve systems of Fredholm and Volterra integro-differential equations of the second kind, higher-order boundary value problems, fractional integro-differential equations and systems of Fredholm and Volterra fractional integro-differential equations of the second kind. Study problems are performed to test the applicability, efficiency and accuracy of this method. The results obtained by Haar wavelet method and Legendre wavelets method in this chapter are compared with the exact solutions and with the results obtained by some other authors, this comparison shows that we obtained better results and more accurate.

Chapter V

In this chapter, we consider the kernels of Baskakov--Durrmeyer and the Szász--Mirakjan--Durrmeyer operators. We establish a Bernstein type inequality for these operators and apply the results to the quasi-interpolants. For the Baskakov--Durrmeyer quasi-interpolants, we give a representation as linear combinations of the original Baskakov--Durrmeyer operators and prove an estimate of Jackson--Favard type and a direct theorem in terms of an appropriate K-functional. Also, we present the Szász--Mirakjan--Durrmeyer operator with respect to an arbitrary measure in the one-dimensional case.

ACKNOWLEDGEMENTS

First of all gratitude and thanks to gracious **Allah** who always helps and guides me. I would like to thank **the prophet Mohamed** "peace be upon him" who urges us to seek knowledge and who is the teacher of mankind.

I am grateful to **Prof. Dr. Ahmed Younis Ghaly**, Professor of Applied Mathematics, Department of Mathematics, Faculty of Education, Ain Shams University, for his help and encouragement during the preparation of the thesis. He taught me many things not only on the technical level but also on the practical and personal levels.

Furthermore; I am thankful and grateful to **Prof. Dr. Hassan Ahmed Zedan**, Professor of Pure Mathematics, Department of Mathematics, Faculty of Science, Kafr El-Sheikh University, for suggesting the topics of the thesis, as well as for his valuable instructions, guidance and continuous follow up on this study. He provided me with his wisdom and knowledge through many discussions.

Also special thanks to my supervisor **Dr. Seham Shabll Tantawy**, lecturer of Mathematics, Department of Mathematics, Faculty of Education, Ain Shams University, for her invaluable comments and illuminating points which were of greatest help to me while preparing the thesis. She offered me much of her precious time and provided me with her knowledge through many discussions.

Thanks also are due to **Prof. Dr. Hassan Agwa,** Head of Mathematics Department, Faculty of Education, Ain Shams University, and all staff members for providing all facilities required to the success of this work.

Finally, my endless appreciation to my kind parents and my beloved family for their support, patience, sacrifices and continuous encouragement.

I owe my mother, my sisters, and my brothers everything.

And last but not least, a very special thanks to my father, may he rest in peace, for all the love, support and encouragement he has always showed me.

Yara Mostafa

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