University College for Women
Department of Mathematics
Ain Shams University

Interactive Approaches for Solving Multicriteria Decision Making Problems

Thesis

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Abbreviations and Notations

1. MDMPs Multiobjective decision making problems.
2. MOP Multiobjective Optimization problem.
3. DM Decision Maker.
4. x Decision Variable.
5. X Feasible Region (Set).
6. $f(x)$ A Vector of m real- valued Functions.
$f(x) = (f_1(x), ..., f_m(x))^T$,
7. $f_i(x)$ Objective Function.
8. $g_s(x)$ Constraint Functions.
9. f^* Ideal Objective Vector.
10. f^{**} Utopian Objective Vector.
11. z Objective Vector.
$z = (z_1, ..., z_m)^T$, $z_i = f_i(x)$
12. Z Feasible Objective Region.
13. U Utility Function.
14. GP Goal Programming.
15. $NLPP$ Nonlinear Programming Problem.
16. P Primal Problem.
17. $K.T.P$ Kuhn -Tucker Problem.
18. $K.T.S.P.P.$ Kuhn –Tucker Saddle Point Problem.
19. D Dual Problem.
20. LD Lagrangian Dual Problem
21. KTCE Kuhn- Tucker Conditions for Efficiency.
22. KTSPCE Kuhn- Tucker Saddle Point Conditions for Efficiency.
23. FMOP Fuzzy Multiobjective Optimization Problem.
24. α -MOP α Multiobjective Optimization Problem.
25. ARP Attainable Reference Point method of Wang et al.
26. RDA Reference Direction Approach of Narula et al.
27. $\bar{f}$ The Attainable Reference Point.
$\bar{f} = (\bar{f}_1, ..., \bar{f}_m)^T$,
28. $MCPP$ Multiobjective Convex Programming Problem.

29. $C^{(1)}$ Class of Functions Which Possess Continuous First Order Partial Derivatives.

30. $S(\bar{x})$ Stability Set of the First Kind Corresponding to the Efficient Solution $\bar{x}$.

31. $Q(\sigma(J))$ The Stability Set of the Second Kind Corresponding to The Side $\sigma(J)$.

32. CCSG Continuous Cooperative Static Game.

33. $G_i(b,\xi)$ The Cost Criterion of Player i .

34. b The State Vector.

35. ξ The Composite Control.

36. Ω Regular Control Constraint Set.

37. FCCSG Fuzzy Continuous Cooperative Static Game.

38. α - CCSG α Continuous Cooperative Static Game.

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Chapter 2: A Unified Approach for Solving Multiobjective Optimization Problems

Chapter 3: Parametric Study for Multiobjective Optimization Problems

Chapter 4: Fuzzy Multiobjective Optimization Problems

Chapter 5: Continuous Cooperative Static Games

Chapter 6: Conclusions and Recommendations for Further Research

Arabic Summary

Abstract

Asmahan Ahmad Khdour. Interactive Approaches for Solving Multicriteria Decision making Problems. Doctor of Philosophy Thesis in Science (PH.D) (Pure Mathematics). University College for Women, Ain Shams University.

The main purpose of this thesis is to study interactive approaches for treating nonlinear multiobjective optimization problems and to introduce a parametric study for these methods.

It is devoted to deduce a unified interactive approach for treating general multiobjective optimization problems (MOPs) with one decision maker and to extend it for treating MOPs under fuzzy environment and to apply it to continuous cooperative static games in both crisp and fuzzy environment.

Also, it is concerned with the characterization of the stability set of the second kind for parametric multiobjective nonlinear convex programming problems, without differentiability assumptions, in both crisp and fuzzy environment and to apply it to continuous cooperative static games in both crisp and fuzzy environment.

Key words: Multiobjective Decision Making, Interactive Approaches, Unified, Convex Programming, Parametric Study, Fuzzy, Cooperative Games.

Summary

Decision making is an integral part of our daily life. It considers situations ranging in complexity from the simple to the most complex involving multiple objectives and literature on this subject produced since 1960s is large as well as diverse in emphasis and style of treatment (see [7]). There has been a growing interest and activity in the area of multiobjective decision making (MDM) in the last years. Modeling and optimization methods have been developed in both crisp and fuzzy environments. Many natural resources management problems, project design problems and financial planning problems in MDM are formulated as multiple objective mathematical programming problems [7], [66] and [35]. In practice, when formulating the multiobjective mathematical programming problem that closely describes and represents the real decision, decision maker (DM) may describe objective functions and constraints in this problem with some vagueness. This vagueness can be efficiently in - terpreted through fuzzy set concept [4] and [56].

Multiobjective optimization methods can be classified according to the DM influence in the optimization process as (see [24]):

1. Methods where DM does not provide information (no-preference methods)

2. Methods where a posteriori information is used (posteriori methods)

3. Methods where a priori information is used (priori methods)

4. Methods where progressive information is used (interactive methods).

Interactive approaches have been invented to combine advantages of both posteriori methods and priori methods and avoid their disadvantages. Since the DM is involved in the entire solution process,

this approach has found better acceptance in practice. Among all the solution approaches, interactive methods have become popular and are considered promising for multiobjective optimization problems (MOPs).

Although numerous interactive procedures have been suggested, none has emerged as a clearly preferred approach (see [62]). Recently, researchers have introduced the concept of the unified algorithm which links various approaches in a way that makes use of the advantages of the linked approaches and avoids their disadvantages (see [75], [52], [16] and [17]).

The stability set approach gives a wide insight for the stability of the solution of parametric nonlinear optimization problems due to a parameter change. The essence of this approach is in the definition, characterization, and determination of a group of parameter sets such as, the set of feasible parameters, the solvability set, and the stability sets of the first and second kinds. These sets were defined and characterized in the crisp environment for parametric nonlinear differentiable programming problems by Osman [41], Osman and Dauer [45] and in fuzzy environment by Osman and El-Banna [46]. In [41], [45] and [46] all the encountered functions are assumed to be in class $C^{(1)}$ (the class of functions which possess continuous first order partial derivatives) on R^n (n- dimensional Euclidean space). Osman [43] has introduced the characterization of the stability set of the first kind for nondifferentiable parametric nonlinear programming problems.

This thesis concerns with interactive approaches for solving MOPs with a parametric study for these methods. It is composed of six chapters:

Chapter one:

It is devoted to review the basic theorems and methods for generating noninferior solutions to MOPs (see [3], [7], [66], [62], and

[35]). A special attention is given to interactive approaches as they are directly connected with the purpose of this thesis. Also it is devoted to review the parametric optimization problem (see [27], [45], [12] and [43]), to present definitions of fuzzy subsets and the concept of α-Pareto optimality (see [4] and [57]) and to review the three basic solution concepts of continuous static games [71].

Chapter two:

It is devoted to deduce a unified interactive approach for solving general MOPs. It combines the advantages of both the Attainable Reference Point method of Wang et al. [74] and the Reference Direction method of Narula et al. [39] and avoids their disadvantages. The results of this chapter are accepted to publication in [50].

Chapter three:

It is concerned with the determination of the stability set of the first kind corresponding to subset of efficient solutions without differentiability and with results concerning the stability set of the second kind developed in [45] and [26]. Also, it is devoted to the characterization of the stability set of the second kind without differentiability for parametric multiobjective nonlinear convex programming problems. Two algorithms for the determination of it in two different spaces are presented. In sections 3.2, and 3.4 differentiability assumptions are not needed. Some results of this chapter are accepted to publication in [48].

Chapter four:

In this chapter the unified interactive approach developed in chapter two is extended for treating MOPs under fuzzy environment. Also, the characterization of the stability set of the second kind for parametric multiobjective convex nonlinear programming problems, without differentiability, under fuzzy environment is introduced. Some results of this chapter are submitted for publication in [49].

Chapter five:

It is devoted to the application of the developed interactive approach to continuous cooperative static games (CCSGs) in both crisp and fuzzy environment and to the application of the stability set of the second kind to CCSGs in both crisp and fuzzy environment.

Chapter six:

It summarizes both the conclusions of this research and recommendations for further research in this field.