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Tanta University

Faculty of Engineering

Physics and Engineering Mathematics Dept.

Nonparametric Tests of Some Classes of Life Distributions

A thesis submitted to

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ABSTRACT

In this Thesis procedures for testing the Exponential Better (Worse) than Used in Average on Specific Interval class of life distribution which written as *EBUASI* (*EWUASI*) are developed. Test statistics are presented based on the following approaches:

1. Total Time on Test Transform (TTT-Transform).
2. Goodness of fit.
3. U-statistic.
4. Moments inequalities.
5. Kernel estimation.

For all developed test statistics, critical values are calculated based on Monte-Carlo simulation, and for some commonly used distributions in reliability power as well as Pitman's asymptotic efficiency are calculated, also Pitman's relative efficiencies are calculated for all approaches except for the Total Time on Test transform; large sample properties and the case of censored data also handled; comparisons between test procedures are made based on the Pitman's relative efficiency.

Key Words: Classes of life distributions, Total Time on Test transform Goodness of fit, U-statistic, Moments inequalities**,** Kernel estimation, Pitman's efficiency and censored data.

المستخلص العربي

في هذه الرسالة تم عمل صيغ اختبارات مختلفة لاختبار الأسية ضد فصل توزيع الحياة الأسى أفضل (أسوأ) من المستخدم في المتوسط على فترة معينة

Exponential Better (Worse) than Used in Average on Specific Interval *EBUASI* (*EWUASI*)

وتم إيجاد إحصاءات اختبار بناء على الطرق التالية:

١. طريقة الاختبار باستخدام تحويل الزمن الكلى Total Time on Test Transform
٢. طريقة الاختبار باستخدام جودة التوفيق Goodness of fit
٣. طريقة الاختبار باستخدام إحصاءه U-statistic U
٤. طريقة الاختبار باستخدام متباينات العزوم Moments inequalities
٥. طريقة الاختبار باستخدام تقدير دالة لب النواة Kernel estimation

وتم حساب القيم الحرجة لهذه الاختبارات في حالة العينات الكاملة والعينات المبتورة عشوائيا وكذلك حساب قوة هذه الاختبارات وذلك لبعض عائلات توزيعات الحياة الشهيرة مثل وايبل، توزيع ماكهاما، توزيع معدل الفشل الخطى.

وتم دراسة خواص الاختبارات من حيث اتساقها وتوزيعها التقريبي.

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Faheem Abd El-Hameed Abass

CONTENTS

List of Tables

List of Figures

List of Symbols

$F(t)$	Cumulative Distribution Function
$\overline{F}(t)$	Survival Function
$\overline{F}_t(x)$	Conditional Survival Function
$f(t)$	Probability Density Function
$R(t)$	Failure Rate (Hazard Rate)
μ	Mean Time to Failure
icv	Increasing Order in Concave
icx	Increasing Order in Convex
$H^{-1}(x)$	Total Time on Test Transform
$\phi_F(x)$	Scaled Total Time on Test Transform
$\overline{X}$	Sample Mean
μ_o, σ_o^2	Mean and Variance under H_o
PAE	Pitman's Asymptotic Efficiency
$\mathbf{e}(\Delta_i, \Delta_j)$	Pitman's Relative Efficiency between Δ_i and Δ_j
K(●)	Kernel Function
H_o	Null Hypothesis
H_1	Alternative Hypothesis
Δ_1	Measure of Departure for the TTT-transform Approach
Δ_2	Measure of Departure for the Goodness of Fit Approach
Δ_3	Measure of Departure for the U-Statistic Approach
Δ_4	Measure of Departure for the Moments Inequalities Approach
Δ_5	Measure of Departure for the Kernel Estimation Approach

$\hat{\Delta}_{1n}$	Empirical Estimation of Δ_1
$\hat{\Delta}_{2n}$	Empirical Estimation of Δ_2
$\hat{\Delta}_{3n}$	Empirical Estimation of Δ_3
$\hat{\Delta}_{4n}$	Empirical Estimation of Δ_4
$\hat{\Delta}_{5n}$	Empirical Estimation of Δ_5
Δ_i^c	Measure of Departure in case of censored data
$\hat{\Delta}_{in}^c$	Empirical Estimation of Δ_i^c

List of Publications

1. Rady, E. A., Lashin, E. F. and Abass, F. A., "Testing *EBUASI* Class of Life Distribution Based on Goodness of Fit Approach". *Interstat, Index for February, No. (3), (2008).*

2. Rady, E. A., Lashin, E. F. and Abass, F. A., "Testing *EBUASI* Class of Life Distribution Based on U-statistic". Accepted for publication in the *Journal of* International *Mathematical Forum*.

3. Rady, E. A., Lashin, E. F. and Abass, F. A., "A Moment Inequality for Exponential Better than Used in Average on Specific Interval Class of Life Distribution with Hypothesis Testing Application". To be appear in the *Journal of Statistical Theory and Applications vol. 7, 2008.*

4. Rady, E. A., Lashin, E. F. and Abass, F. A., "Testing Exponential Better than Used in Average on Specific Interval Class of Life Distribution Based on Total Time on Test Transform". Submitted for publication.

5. Rady, E. A., Lashin, E. F. and Abass, F. A., "Testing Exponential Better than Used in Average on Specific Interval Class of Life Distribution Based on Kernel Method". Submitted for publication.

CHAPTER I

Introduction and Review of Literatures

The word reliability is associated with the civilization of mankind to compare one item/person with another. Trustworthy, dependable and consistent are the words, which can be used to give an indication of why the characteristic of reliability is valued. Reliability cannot be precisely measured with respect to human behavior but can give an indication that a particular person is more reliable than the other. The judgment can be easier when it is associated with human behavior or function. For example, the degree of punctuality of individuals for attending their work place or attending their work without failure can be used to measure their reliability in performing a particular function.

In case of man made products/systems the role of design engineer becomes more crucial to eliminate/minimize the large scale failures to avoid accidents in case of automotive vehicles etc. where risk of human life is involved.

In fact, the characteristic of reliability is usually used to describe some function or task or in widest sense, it may be said to be a measure of performance. A person who completes his work in time is said to be more reliable than the other who does not. Now it is easy to state, that the concept of reliability is not only associated with human behavior or activity but can also be applied to other inventions of the man kind, by directly measuring

their performance or by knowing the failure rates of the equipments/systems. The growing awareness of reliability arises from the fact that there is a need for efficient, economic and continuous running of equipment/system in any organization for achieving the targeted production at a minimum cost to face the present competitive world.

In World War II the need for reliability was felt because of failure of many military operations in spite of the best efforts from the users. After World War II a study was conducted which revealed the following facts:

1. The electric equipments/systems used by the Navy were operative for only *30%* of its total available time because of frequent failures/maintenance problems.
2. Army equipments were either under repair/breakdown for almost *60-75%* of its total time, which again created problems for a successful mission.
3. Air-force study conducted for about *5* years shown that the maintenance costs were very high and even sometimes many fold of the original cost of the equipments/systems due to frequent breakdowns.

The above facts may be the reason that during war period the availability of equipments is of prime importance besides its cost. With the above information investigations were further carried out and a concept of reliability group/engineer came into existence. Since then efforts are continuing in this are to achieve the desired goals in industrial organizations. The concept of reliability in the minds of people/engineers in general has