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**IN- PLANE STABILITY CONSIDERATIONS OF
TRAPEZOIDAL STEEL FRAMEWORKS.**

By

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CONTENTS

Notations	Page i
List of Figures	ii
Chapter 1: Introduction	
1.1 Introduction	1
1.2 Statement of the Problem	2
1.3 Behavior of Structures Under Axial Compressive Loads	3
1.4 Thesis Arrangement	4
 Chapter 2: Review of Previous work & Methods of Analysis	
2.1 Previous Research in the Field of Elastic Buckling	6
2.2 Methods of Analysis of Elastic Buckling	17
2.2.1 Introduction	17
2.2.2 Assumptions	17
2.2.3 Direct Method	18
Operations of Rotation and Sway	18
The General State of Sway	22
Addition Rules of Sway States	22
Stability Equations	24
Formation of the Stability Equations	25
Stability Conditions	25
2.2.4 Stiffness Method of Analysis	25
Global and Local Axes	26
Element Stiffness Formulation	27
Stiffness Matrix for a Frame	28
2.2.5 Computer Programs	29
2.2.6 Accuracy of Results	29
 Chapter 3: Elastic Stability of Rectangular Frames with Corner-to-Column Cross Bracings	
3.1 Introduction	30
3.2 Rectangular Frames with Fixed Base.	

	Page
3.2.1 Frames with Axially Stiff Cross Bracings in Sway Mode	30
3.2.2 Frames with Axially Stiff Cross Bracings in Symmetric Mode	34
3.2.3 Frames with Axially Elastic Cross Bracings in Sway Mode	37
3.2.4 Frames with Axially Elastic Cross Bracings in Symmetric Mode	41
3.2.5 Discussion of Results	46
3.3 Rectangular Frames with Corner-Column Cross Bracing and Hinged Base.	
3.3.1 Frames with Axially Stiff Cross Bracings in Sway Mode	53
3.3.2 Frames with Axially Stiff Cross Bracings in Symmetric Mode	55
3.3.3 Frames with Axially Elastic Cross Bracings in Sway Mode	57
3.3.4 Frames with Axially Elastic Cross Bracings in Symmetric Mode	61
3.3.5 Discussion of Results	65
Chapter 4: Elastic Stability of Rectangular Frames with Intermediate Central Cross Braced Columns	
4.1 Introduction	71
4.2 Rectangular Frames with Cross Bracing and Fixed Base.	
4.2.1 Frames with Axially Stiff Cross Bracings in Sway Mode	71
4.2.2 Frames with Axially Stiff Cross Bracings in Symmetric Mode	75
4.2.3 Frames with Axially Elastic Cross Bracings in Sway Mode	77
4.2.4 Frames with Axially Elastic Cross Bracing in Symmetric Mode	80
4.2.5 Representation and Discussion of Results	84
4.3 Rectangular Frames with Cross Bracing and Hinged Base.	

	Page
4.3.1 Frames with Axially Stiff Cross Bracings in Sway Mode	91
4.3.2 Frames with Axially Stiff Cross Bracings in Symmetric Mode	94
4.3.3 Frames with Axially Elastic Cross Bracings in Sway Mode	95
4.3.4 Frames with Axially Elastic Cross Bracings in Symmetric Mode	98
4.3.5 Representation and Discussion of Results	102
 Chapter 5: Best Proportioning of Rectangular Frames with Intermediate Column Cross Bracings	
5.1 Introduction	108
5.2 Rectangular Frames with Intermediate Column Bracings and Fixed Bases	
5.2.1 Frames with Horizontal Equivalent Rigid Bracing Beam in Anti-Symmetric Mode	108
5.2.2 Frames with Axially Stiff Cross Bracings in Sway Mode	110
5.2.3 Frames with Axially Stiff Cross Bracings in Symmetrical Mode	112
5.2.4 Frames with Axially Elastic Cross Bracings in Sway Mode	114
5.2.5 Frames with Axially Elastic Cross Bracings in Symmetrical Mode	117
5.2.6 Presentation and Discussion of Results	122
5.2.7 Cases of Different Span to Height Ratios	125
5.3 Rectangular Frames with Intermediate Column Bracings and Hinged Bases	
5.3.1 Frames with Rigid Beam in Anti-Symmetrical Mode	144
5.3.2 Frames with Axially Stiff Cross Bracings in Sway Mode	146
5.3.3 Frames with Axially Stiff Cross Bracings in Symmetric Mode	148
5.3.4 Frames with Axially Elastic Cross Bracings in Sway Mode	150
5.3.5 Frames with Axially Elastic Cross Bracings in Symmetric Mode	152
5.3.6 Presentation and Discussion of Results	155
5.2.7 Cases of Different Span to Height Ratios	157

Chapter 6:	Elastic Stability of Braced Trapezoidal Frames	Page 165
6.1	Introduction	165
6.2	Trapezoidal Frames with Horizontal Link Members and Fixed Bases in Sway Mode	165
6.2.1	Frames with One Horizontal Link Members	165
6.2.2	Frames with Two Horizontal Link Members	169
6.2.3	Frames with Three Horizontal Link Members	173
6.2.4	Frames with Two Horizontal Link Members Randomly Spaced	175
6.2.5	Trapezoidal Frame with Rigid Beam	177
6.2.6 TO 6.2.8	Analytical Comparisons of Trap. Frames with Horizontal Bracing	179
6.2.9	Results and Discussion	185
6.3	Trapezoidal Frames with Corner to Column Cross Bracings and Hinged Bases	187
6.3.1	Frames with Axially Stiff Cross Bracings in Sway Mode	187
6.3.2	Frames with Axially Stiff Cross Bracings in Symmetric Mode	190
6.3.3	Frames with Elastic Cross Bracings in Sway Mode	192
6.3.4	Frames with Elastic Cross Bracing in Symmetric Mode	202
6.3.5	Presentation and Discussion of Results	208
6.4	Trapezoidal Frames with Corner to Column Cross Bracings and Fixed Bases	226
6.4.1	Frames with Axially Stiff Cross Bracings in Anti-Symmetric (Sway) Mode	226
6.4.2	Frames with Axially Stiff Cross Bracings in Symmetric Mode	228
6.4.3	Frames with Elastic Cross Bracings in Anti-Symmetric (Sway) Mode	230
6.4.4	Frames with Elastic Cross Bracings in Symmetric Mode	233
6.4.5	Presentation of Results	239
6.4.6	Discussion of the Results	241

	Page
Chapter 7: Miscellaneous Framework Systems	
7.1 Frames with Horizontal Bracing	243
7.2 Multi-Story Rectangular Frames with Hinged Bases	244
7.3 Rectangular Frames with Cross Bracings	250
7.4 Trapezoidal Frames	255
Chapter 8: Summary, Conclusions & Recommendations	
8.1 Summary	260
8.2 Conclusions	260
8.3 Recommendations for Future Work	261
Appendix 1 References	A-1
Appendix 2 Geometrical Properties of Trapezoidal Frames	A-7
Appendix 3 General State of Sway	A-10

NOTATIONS

The following symbols are used in the present thesis; any other symbols not stated below are defined when they are first used.

A, B, C, D	: frame joints of un-deformed frame;
A_1, B_1, C_1, D_1	: frame joints of deformed frame;
B'	: bracing stiffness dimensionless constant;
F	: force in tie member;
H	: secondary horizontal reaction;
I	: second moment of inertia;
k	: member bending stiffness (EI/L);
L	: column length;
L_b	: beam length;
L_T	: Length of bracing member;
P	: external normal force on column;
P_E	: Euler's buckling load ($\pi^2 EI / L^2$)
P_{cr}	: critical buckling load;
ρ	: ratio of axial load to Euler's load, (P / P_E);
ρ_{cr}	: critical ratio of axial load to Euler's load, (P / P_E);
s, c, m, n, o	: stability functions;
a	: length ratio between bracing joints to column length "L"
δ	: deflection perpendicular to column;
α	: joint secondary rotation;
φ	: column inclination to the vertical;
ψ	: bracing inclination to the horizontal;
h	: length ratio between, corner to bracing joint, to column length "L";
g	: length ratio between, lower base to bracing joints, to column length "L";
B	: bracing coefficient factor;
$[D]$	: member displacement vector;
$\{F\}$	: load vector;
$[K]$	: stiffness matrix;
$[T]$	: transformation matrix;
$[T]^t$	: transpose of the transformation matrix;
$[\bar{K}]$	: global stiffness matrix;
x, y	: coordinates;
μL	: stability dimensionless parameter ($\sqrt{\pi^2 \rho}$)

LIST OF FIGURES

INTRODUCTION		Page
Fig. (1-1):	Load deflection behavior of frames.	2
 REVIEW OF PREVIOUS WORK AND METHOD OF ANALYSIS		
Fig. (2.1):	Euler's Elastic curves	8
Fig. (2.2):	The critical Load P_{cr} , and the Accurate Load P_1	9
Fig. (2.3):	Operations of Rotation for Member prevented From Side Sway	19
Fig. (2.4):	Operations of Translation for Members Permitted to Sway	20
Fig. (2.5):	Superposition of States of Sway for members with Fixed Ends	23
Fig. (2.6):	Superposition of States of Sway for members with Hinged Ends	24
Fig. (2.7):	Resolution of General State of Sway of Intermediate Column In Multistory Frame.	24
Fig. (2.8):	Local and Global Axes.	27
Fig. (2.9):	Stiffness Types Used in Stiffness Method.	27
 OPERATIONS OF SWAY AND ROTATION FOR MEMBERS		
 RECTANGULAR FRAMES WITH CORNER TO COLUMN BRACINGS AND FIXED BASES		
Fig. (3.1):	Frames with Axially Stiff Bracings in Sway Mode.	32
Fig. (3.2):	Frames with Axially Stiff Bracings in Symm. Mode.	35
Fig. (3.3):	Frames with Axially Elastic Bracings in Sway Mode.	38
Fig. (3.4):	Frames with Axially Elastic Bracings in Symmetrical Mode.	42
 RECTANGULAR FRAMES WITH CORNER TO COLUMN BRACINGS AND HINGED BASES		
Fig. (3.11):	Frames with Axially Stiff Bracings in Sway Mode.	54
Fig. (3.12):	Frames with Axially Stiff Bracings in Symm. Mode.	56
Fig. (3.13):	Frames with Axially Elastic Bracings in Sway Mode.	58
Fig. (3.14):	Frames with Axially Elastic Bracings in Symmetrical Mode.	62

	Page
RECTANGULAR FRAMES WITH INTREMEDATE CENTRAL BRACINGS AND FIXED BASES	
Fig. (4.1):	Frames with Axially Stiff Bracings in Sway Mode. 72
Fig. (4.2):	Frames with Axially Stiff Bracings in Symm. Mode. 76
Fig. (4.3):	Frames with Axially Elastic Bracings in Sway Mode. 78
Fig. (4.4):	Frames with Axially Elastic Bracings in Symmetrical Mode. 81
RECTANGULAR FRAMES WITH INTREMEDATE CENTRAL BRACINGS AND HINGED BASES	
Fig. (4.11):	Frames with Axially Stiff Bracings in Sway Mode. 92
Fig. (4.12):	Frames with Axially Stiff Bracings in Symmetrical mode. 94
Fig. (4.13):	Frames with Axially Elastic Bracings in Sway Mode. 96
Fig. (4.14):	Frames with Axially Elastic Bracing in Symmetrical mode. 99
RECTANGULAR FRAMES WITH INTER-BRACED COLUMNS AND FIXED BASES	
Fig. (5.1):	Frames with Horizontal Rigid Beam. 109
Fig. (5.2):	Frames with Axially Stiff Bracings in Sway Mode. 111
Fig. (5.3):	Frames with Axially Stiff Bracings in Symmetrical Mode. 112
Fig. (5.4):	Frames with Axially Elastic Bracings in Sway Mode. 115
Fig. (5.5):	Frames with Axially Elastic Bracings in Symmetrical mode. 118
RECTANGULAR FRAMES WITH INTER-BRACED COLUMNS AND HINGED BASES	
Fig. (5.23):	Frames with Rigid Beam in Sway Mode. 145
Fig. (5.24):	Frames with Axially Stiff Bracings in Sway Mode. 146
Fig. (5.25):	Frames with Axially Stiff Bracings in symmetrical mode. 149
Fig. (5.26):	Frames with Axially Elastic Bracings in Sway Mode. 150
Fig. (5.27):	Frames with Axially Elastic Bracings in symmetrical mode. 153
TRAPEZOIDAL FRAMES WITH HORIZONTAL BRACINGS AND FIXED BASE IN SWAY MODE	
Fig. (6.1):	Frames with One Horizontal Bracing. 166

	Page
Fig. (6.2): Frames with Two Horizontal Bracings in Sway Mode	170
Fig. (6.3): Frames with Three Horizontal Bracings in Sway Mode.	173
Fig. (6.4): Frames with Randomly Spaced Two Horizontal Bracings in Sway Mode.	175
Fig. (6.5): Trapezoidal Frame with Rigid Beam.	177
Fig. (6.6): Trapezoidal Frame with Rigid Beam and Horizontal bracing.	179
Fig. (6.7): Trapezoidal Frame with Upper Link Beam	181
Fig. (6.8): Trapezoidal Frame with Upper Link Beam and Horizontal Bracing..	183

TRAPEZOIDAL FRAMES WITH CROSS BRACINGS AND HINGED BASES

Fig. (6.10): Frames with Axially Stiff Bracings in Sway Mode.	188
Fig. (6.11): Frames with Axially Stiff Bracings in symmetrical Mode.	191
Fig. (6.12): Frames with Axially Elastic Bracings in Sway Mode.	195
Fig. (6.13): Frames with Axially Elastic Bracings in Symmetrical Mode.	203

TRAPEZOIDAL FRAMES WITH CROSS BRACINGS AND FIXED BASES

Fig. (6.30): Frames with Axially Stiff Bracings in Sway Mode.	226
Fig. (6.31): Frames with Axially Stiff Bracings in Symmetrical Mode.	228
Fig. (6.32): Frames with Axially Elastic Bracings in Sway Mode.	231
Fig. (6.33): Frames with Axially Elastic Bracings in Symmetrical Mode.	233

RESULTS AND RELATIONSHIPS (VALUES OF CRITICAL LOADS)

RECTANGULAR FRAMES WITH CORNER TO COLUMN BRACINGS AND FIXED BASES

Fig. (3.5): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic Bracings ($K_b/K=0.0$).	47
Fig. (3.6): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.25$).	48
Fig. (3.7): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.50$)	49

	Page
Fig. (3.8): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=1.00$).	50
Fig. (3.9): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=4.00$).	51
Fig. (3.10): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=\infty$).	52

RECTANGULAR FRAMES WITH CORNER TO COLUMN BRACINGS AND HINGED BASES

Fig. (3.15): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.0$).	66
Fig. (3.16): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=1.00$).	67
Fig. (3.17): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=4.00$).	68
Fig. (3.18): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=\infty$).	69
Fig. (3.19): Relation Between P_{cr}/P_E and α with Axially Stiff Bracing for different K_b/K ratios.	70

RECTANGULAR FRAMES WITH INTREMEDATE CENTRAL BRACINGS AND FIXED BASES

Fig. (4.5): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.0$).	85
Fig. (4.6): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.25$).	86
Fig. (4.7): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.50$).	87
Fig. (4.8): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=1.00$).	88
Fig. (4.9): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=4.00$).	89
Fig. (4.10): Relation Between P_{cr}/P_E and α for frames with Axially Elastic bracing ($K_b/K=\infty$).	90

RECTANGULAR FRAMES WITH INTREMEDATE CENTRAL BRACINGS AND HINGED BASES

Fig. (4.15): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=0.0$).	103
Fig. (4.16): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=1.00$).	104

	Page
Fig. (4.17): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K=4.00$).	105
Fig. (4.18): Relation Between P_{cr}/P_E and α for Frames with Axially Elastic bracing ($K_b/K= \infty$).	106
Fig. (4.19): Relation Between P_{cr}/P_E and α for Frames with Axially Stiff Bracing in different K_b/K ratio.	107

RECTANGULAR FRAMES WITH INTER-BRACED COLUMNS AND FIXED BASES

Fig. (5.6): Relation Between P_{cr}/P_E and h for Frames with Rigid Horizontal Beam in Different K_b/K Ratio.	127
Fig. (5.7): Relation Between P_{cr}/P_E and h for Frames with Axially Stiff Bracing ($K_b/K=0.0$).	128
Fig. (5.8): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=0.0$) , ($AL^2/I = 100$).	129
Fig. (5.9): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=0.0$) , ($AL^2/I = 400$).	130
Fig. (5.10): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=0.0$) , ($AL^2/I = 1000$).	131
Fig. (5.11): Relation Between P_{cr}/P_E and h for Frames with Axially Stiff Bracing ($K_b/K=0.50$).	132
Fig. (5.12): Relation Between P_{cr}/P_E and h for Frames with Axially Stiff Bracing ($K_b/K=1.00$).	133
Fig. (5.13): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=1.00$) , ($AL^2/I = 100$).	134
Fig. (5.14): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=1.00$) , ($AL^2/I = 400$).	135
Fig. (5.15): Relation Between P_{cr}/P_E and h for Frames with Axially Stiff Bracing ($K_b/K=4.00$).	136
Fig. (5.16): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=4.00$) , ($AL^2/I = 100$).	137
Fig. (5.17): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=4.00$) , ($AL^2/I = 400$).	138
Fig. (5.18): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=4.00$) , ($AL^2/I = 1000$).	139
Fig. (5.19): Relation Between P_{cr}/P_E and h for Frames with Axially Stiff Bracing ($K_b/K= \infty$).	140
Fig. (5.20): Relation Between P_{cr}/P_E and h for Frames with Axially Elastic Bracing ($K_b/K=\infty$) , ($AL^2/I = 100$).	141
Fig. (5.21): Relation Between P_{cr}/P_E and h for frames with	Page 142