

**A computational Study of the external forces  
effects on the motion of fluids through  
biological tissues**

**A Thesis**

**Submitted as a Partial Fulfillment for the Requirement of the  
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(Applied Mathematics)**

To

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**Degree** : master degree in teacher preparation in science  
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# Acknowledgments

## Acknowledgments

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# Nomenclature

## **Nomenclature:**

$u$	velocity in x direction
$v$	velocity in y direction
$x$	space coordinate
$\omega_b$	Blood perfusion rate
$b$	blood
$T$	temperature
$H_0$	Magnetic field in free space
$\bar{J}$	Current density
$E_0$	electric field in free space
$L$	distance
$v_r$	Radial velocity
$v_\theta$	Tangential velocity
$v_z$	Axial velocity
$r$	Radial axis
$z$	Vertical axis
$P$	Pressure
$\theta$	Tangential axis
$C$	Concentration
$P_r$	Prandtl number, $= \mu c_p / k$
$\eta$	Normal distance from the disk, $= z(\Omega / \nu)^{1/2}$
$\omega$	Angular velocity
$\mu$	Dynamic viscosity
$k$	thermal conductivity coefficient

$\rho$	fluid density
$\nu$	kinematic viscosity
$M$	porosity parameter $M = \nu / k\omega$
$k_T$	Thermal diffusion ratio
$\tau$	Stress tensor
$S_r$	Shoret number
$D_f$	Dufour number $= D k_T / \nu$
$D$	Molecular diffusivity
$k_r$	Thermal diffusion
$S_c$	Shmidt number
$\chi$	stream function
$f$	similarity dependent variable
$Re$	Reynolds number, $Re = u \cdot L / \nu$
$S$	dimensionless wall shear stress
$t$	time variable
$T$	temperature
$x, y$	Cartesian coordinates
$\Theta$	dimensionless temperature
$\phi$	dimensionless cocentration
$c_p$	specific heat capacity
$\beta_c$	coefficient of the thermal expansion with concentration
$T_m$	mean fluid temperature
$\beta$	coefficient of the thermal expansion

# Summary



## **Summary**

Applied mathematics to biological systems gives the ability to construct mathematical models. Such models are mathematical systems that attempt to represent the complex interactions of biological systems in a way simple enough for their consequences to be understood and explored. Traditionally models that allowed biologists to see a problem in a simplified way have been complicated structure, while mathematical models that constructed to exhibit simple biological properties that could be analyzed. This kind of model, however, is restricted by technology as well as technological ingenuity. Mathematical models have no such restriction and can be used to construct any sort of biological system; respiratory flows, pulsating blood flow, micro- and macro-circulation systems bioheat and mass transfer models are some examples of these mathematical models these flows may be studied under a well-known branch of science it is named by biofluid mechanics. By biofluid mechanics we can understand the physiological processes that occur in the human circulation and analyze the physical mechanisms that under line them. Understanding the basic processes occurring in the human body will facilitate the engineering design and construction of new medical devices and machinery. Our thesis concerns with A computational Study of the external forces effects on the motion of fluids through biological tissues such as blood flow through tissues.

The present thesis consists of four chapters with tow summaries one of them with Arabic language and the other with English language and list of

references for books and papers related to the subjects of the thesis.

## **Chapter ١**

chapter one includes introduction about general concepts of Fluid Mechanics , Newtonian fluid and Non-Newtonian fluid, magneto-hydrodynamics, and basic equations, porous medium, porosity, Darcy law and non- Darcian equations.

We summarized the basic equations of Newtonian and non- Newtonian fluids (continuity equation, momentum equation, Energy equation, and Concentration equation), models of heat transfer (radiation, convection, conduction, evaporation), bioheat and mass transfer.

Finally we introduce analytical solution of non-linear partial differential equations such as homotopy perturbation methods, homotopy perturbation methods.

## **Chapter ٢**

The second chapter investigated the effects of electromagnetic field on the blood flow through human finger with heat and mass transfer. The momentum, heat and mass transfer and bioheat equations which govern our phenomena are solved by using linear transformation method. The forms of velocity, temperature, concentration and bioheat transfer distribution are obtained analytically. The effects of the problem parameters on these formulas are illustrated graphically by a set of figures.

## **Chapter 3**

In chapter three, unsteady fluid motion adjacent to stretching surface with heat and mass transfer has been investigated. The nonlinear partial differential equations governing the problem have been solved by using the similarity transformation and homotopy perturbation methods. The velocity, temperature and concentration distributions have been obtained analytically and illustrate graphically through a set of figures. The effect of different parameters of the problem on these distributions is discussed.

## **Chapter 4**

chapter four includes a mathematical analysis of the steady flow of an incompressible viscous non-Newtonian fluid through an infinite rotating porous disk in a porous medium with heat and mass transfer. The nonlinear partial differential equations governing the problem have been transformed to ordinary differential equations by using the Von Karman transformation. The homotopy perturbation method was implemented to obtain analytical solution for ordinary system. The distributions of velocity, temperature and concentration are illustrated graphically for different parameters.

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