Ain shams University
College of Women for Arts,
Science and Education

Solutions of Dynamic Equations

Thesis

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(Pure Mathematics)
By

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Nesreen, Yaseen

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Dedicated To

The blessed soul of my late grandmother

My late father whom I have never seen.

My beloved mother.

My dear husband Mohamed.

My beloved son Yehya

My beloved daughter Jodi.

My sweetest sister Noha.

My dear uncle Said

My love Nourseen and Ahmed

Contents

Abstract

Name: Nesreen Abd El Hamed Abd El Hamed Yaseen

Title: solutions of dynamic equations.

Ph. D. of scince in Pure Mathematics, University College for Women, Ain Shams University.

The main purpose of this thesis is to investigate the Hyers-Ulam and Hyers-Ulam-Rassias stability of abstract linear dynamic equations on time scales.

Also, we investigate the Hyers-Ulam-Rassias stability of Volterra integral dynamic equation on time scales.

Key Words: Dynamic equations on time scales, Hyers-Ulam stability, Hyers-Ulam-Rassias stability, Volterra integral equations on time scales.

Summary

A time scale $\mathbb{T}$ is an arbitrary non-empty closed subset of the set of real number. The object of the theory of dynamic equations on time scales is to unify continuous and discrete calculus which was introduced by Stefan Hilger in his Ph .D. in 1988 [29]. The theory presented a structure where, once a result is established for general time scale, special cases include a result for differential equations and a result for difference equations. A great deal of work has been done since 1988 unifying the theory of differential equations and the theory of difference equations by establishing the corresponding results in time scale setting. Recent two books of Bohner and Paterson [8,9] provide both an excellent introduction to the subject and up to-date coverage of much of the theory . Studying the theory of dynamic equations on time scales has attracted many authors in the last few years, because of the wide applications in engineering, industry, biology, economics and other field, see [5]. Also, the stability analysis of dynamic equations has become an important topic

both theoretically and practically because dynamic equations occur in many areas such as mechanics, physics, and economics.

This work focuses on the Hyers-Ulam and Hyers-Ulam-Rassias stability of dynamic equations on time scales . In 1940 Ulam [63] posed his problem of stability which solved by Hyers [30], and the result of Hyers was generalized by Rassias [58]. Alsina and Ger [1] were the first authors who investigated the Hyers-Ulam stability of a differential equation. Also of interest, that many of articles were edited

by Rassias [57], dealing with Ulam, Hyers-Ulam and Hyers-Ulam-Rassias stability . Many papers introduced the Hyers-Ulam and Hyers-Ulam-Rassias stability for differential equations and integral equations[15,37]. On the other hand side the papers which were presented the Hyers- Ulam and Hyers-Ulam-Rassias stability of dynamic equations and integral dynamic equation on time scales are still very few, may be except the studies which were presented in the papers [4,7].

This thesis investigates the (Hyers-Ulam-Rassias) stability for first and second order abstract dynamic equations on time scales. Also, in this thesis we investigate the Hyers-Ulam-Rassias stability of integral dynamic equation on time scale.

The thesis is organized as follows:

Chapter I:

In this Chapter, we introduce the basic concepts and terminology of time scales calculus. We introduce basic theorems of delta derivative and delta integral which were presented in Bohner [10]. The exponential function and the operator exponential function on time scales with some properties of them are given. Also, we introduce the concept of a rd-continuous matrix, and a regressive matrix valued functions. Also this chapter presents definitions and some properties of operator exponential function $e_A(t,s)$ in Banach spaces. Moreover, basic concepts of the theory of C_0-Semigroup of bounded linear operators are introduced . Also some properties of a $\mathbf{C_0}$-Semigroup and its generator which were established in [2] are introduced in this chapter.

Chapter II

In this chapter we introduce the papers which presented the Hyers-Ulam and Hyers-Ulam-Rassias Stability of differential equation. Also, we introduce the papers which proved the Hyers-Ulam of integral equations. This chapter also, presents the articles dealing with the Hyers-Ulam stability of dynamic equations with constant coefficients on time scales[7]

Chapter III

In this chapter we investigate the Hyers-Ulam Stability of the abstract dynamic equation of the form

$$x^{\Delta}(t) = A(t)x(t) + f(t), t \in \mathbb{T}, \qquad x(t_0) = x_0 \in \mathbb{X}, \qquad (1)$$

where $A: \mathbb{T} \to L(\mathbb{X})$ (The space of all bounded linear operators from a Banach space $\mathbb{X}$ into itself) and f is rd-continuous from a time scale $\mathbb{T}$ to $\mathbb{X}$. This chapter deal with two cases, the first case if $A(t)$ is regressive, and the second case if $A(t)$ is not regressive. Some examples illustrate the applicability of the main results.

The results of this chapter were published in [4].

Chapter IV

This chapter is devoted to establishing Hyers-Ulam Stability of the abstract dynamic equation of the form

$$x^{\Delta\Delta}(t) + A(t)x^{\Delta}(t) + R(t)x(t) = f(t), \quad t \in \mathbb{T}, \tag{2}$$

where $A, R: \mathbb{T} \to \mathrm{L}(\mathbb{X})$, the space of all bounded linear operators from a Banach space $\mathbb{X}$ into itself, and f is rd-continuous from a time scale $\mathbb{T}$ to $\mathbb{X}$. Some examples illustrate the applicability of the main result.

The results of this chapter have been prepared in an article which was submitted and accepted in Journal: International Journal of Math. Anal. [5].

Chapter V

In this chapter we establish the Hyers-Ulam-Rassias stability for Equations (1) and (2).

Also, some examples illustrate the applicability of the main result.

Chapter VI

In this chapter we investigate the Hyers-Ulam-Rassias stability for the Volterra integral equation of the form

$$y(t) = \int_{t_0}^{t} f\big(t, s, y(s\,)\big)\,\Delta s, \qquad t \in \mathbb{T}$$

Chapter VII

This chapter contains the conclusions and some points for future research.

Notations

$\mathbb{N}$:= { 1,2,…..}
$\mathbb{N}_0 := \mathbb{N} \cup \{0\}$
$\mathbb{Z}$ is the set of all integer numbers.
$\mathbb{Q}$ is the set of rational numbers.
$\mathbb{R}$ is the set of all real numbers.
$\mathbb{C}$ is the set of complex numbers.
$\mathbb{T}$ is a time scale (i.e. a nonempty closed subset of $\mathbb{R}$).
$[a,b]_{\mathbb{T}} := [a,b] \cap \mathbb{T}$ for $a,b \in \mathbb{T}$ (a closed interval of $\mathbb{T}$).
σ is the forward jump operator on $\mathbb{T}$.
ρ is the backward jump operator on T.
μ is the graininess function on $\mathbb{T}$.
$\mathcal{R}$ is the family of regressive functions.
C_{rd} is the family of right dense continuous functions.
f^{Δ} is the delta derivative ($\Delta -$derivative) of a function *f*.
$\mathbb{X}$ is a Banach space.
L($\mathbb{X}$) is the set of all bounded linear operators from a Banach space $\mathbb{X}$ into itself.
D(*A*) is the domain of *A*.

Chapter I

The Time Scales Calculus

1.1 Introduction

The time scales calculus is a fairly new area of research. It was introduced in 1988 by Stefan Hilger in his Ph .D. Thesis [29] supervised by Bernd Aulbach, see also [8,9]. It combines the traditional areas of continuous and discrete analysis into one theory. In this chapter we give several foundational definitions and notations of basic calculus of time scales introduced in the excellent recent books by M. Bohner and Paterson [10,11]. The object of the theory of dynamic equations on time scales is to unify continuous and discrete calculus which was introduced by Stefan Hilger [29], in his Ph .D. thesis.

1.2 Basic Definition and Terminology

Definition 1.2.1. [10] A time scale $\mathbb{T}$ is an arbitrary non-empty closed subset of the set of real numbers.

Definition 1.2.2. [10]*The mappings* $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ *is defined by* $\sigma(t) = inf\{s \in \mathbb{T}: s > t\}$, *and* $\rho(t) = sup\{s \in \mathbb{T}: s < t\}$,*are called the jump operators*.

Definition 1.2.3.[10] *A point $t \in \mathbb{T}$ is said to be right–dense if $\sigma(t) = t$, right-scattered if $\sigma(t) > t$, left-dense if $\rho(t) = t$, left-scattered if $\rho(t) < t$, isolated if $\rho(t) < t < \sigma(t)$, and dense if $\rho(t) = t = \sigma(t)$.*

A time scale is said to be discrete if t is left-scattered and right scattered at the same time for all $t \in \mathbb{T}$ and it is called continuous if t is right-dense and left-dense at the same time for all $t \in \mathbb{T}$. For every $a, b \in \mathbb{T}$, $a \leq b$, we define the interval $[a, b]_{\mathbb{T}}$ in $\mathbb{T}$ by $[a, b]_{\mathbb{T}} \coloneqq [a, b] \cap \mathbb{T}$.

Definition1. 2.4. [10] *Let $t \in \mathbb{T}$, the graininess function $\mu: \mathbb{T} \to [0, \infty[$ is defined by*

$$\mu(t) = \sigma(t) - t.$$

Example 1.2.1 Consider the cases $\mathbb{T} = \mathbb{R}$ and $\mathbb{T} = \mathbb{Z}$.

(i) If $\mathbb{T} = \mathbb{R}$, then for any $t \in \mathbb{R}$

$$\sigma(t) = \inf\{s \in \mathbb{R} : s > t\} = \inf(t, \infty) = t$$

and

$\rho(t) = \sup\{s \in \mathbb{R} : s < t\} = \sup(-\infty, t) = t$. Hence every point $t \in \mathbb{R}$ is dencse. The graininess function μ turns out to be

$\mu(t) = 0$ for every $t \in \mathbb{R}$.

(ii) If $\mathbb{T} = \mathbb{Z}$, then we have for any $t \in \mathbb{Z}$

$$\sigma(t) = \inf\{s \in \mathbb{Z} : s > t\} = \inf\{t+1, t+2+t+3, \dots\} = t+1,$$

$$\rho(t) = \sup\{s \in \mathbb{Z} : s < t\}$$

$$= \sup\{\dots, t-3, t-2, t-1\} = t-1,$$

The graininess function μ turns out to be

$\mu(t) = 1$ for every point $t \in \mathbb{Z}$.

1.3 Differentiation

In this section we present the notion of differentiability of a function f on time scales

Definition 1.3.1.[10] *A function $f: \mathbb{T} \to \mathbb{X}$ is called regulated provided its right-sided limits exist (finite) at all right-dense points in $\mathbb{T}$ and its left-sided limits exist (finite) at all left-dense points in $\mathbb{T}$.*

Definition 1.3.2.[10] *A function $f: \mathbb{T} \to \mathbb{X}$ is called rd-continuous provided*

(i) f is continuous at every right –dense point;

(ii) $\lim_{s \to t^-} f(s)$ exists (finite) at left-dense points in $\mathbb{T}$.

The set of rd-continuous functions $f: \mathbb{T} \to \mathbb{X}$ will be denoted by $C_{rd}(\mathbb{T}, \mathbb{X})$.

Definition1.3.3 (The Delta Derivative)[10]. *A function $f: \mathbb{T} \to \mathbb{X}$ is called Δ-differentiable at $t \in \mathbb{T}^k$ if there exists an*