شكر وتقدير

بداية أشكر من تفضل وتكرم، وأعطى وأنعم، ووفق ويسر، خالقي ورازقي وولي نعمتي (ربي) ورب كل شيء.

فالحمد لله الذي بنعمته تتم الصالحات، فلقد وفقني الله سبحانه بفضله وجوده ومنه وكرمه في إنجاز هذه الرسالة. فالحمد لله على الدوام وله الشكر على التمام، فهو: "الرَّحْمَنُ .عَلَّمَ القُرْآنَ . خَلَقَ الإِنسَانَ .عَلَّمَهُ البَيَانَ " الرحمن.(1-4)

والصلاة والسلام على سيد الأنام ، وحبيب الرحمن سيدنا محمد - صلى الله عليه وسلم- من حثنا على طلب العلم فهو معلم البشرية فصلاة وسلاماً عليه ننال بهما في الدنيا عزة وكرامة ، وفي الآخرة صحبة وشفاعة.

كما لا يسعني أن أتقدم بعميق الشكر والإمتنان لهيئة الإشراف وهم :

الأستاذ الدكتور/ صبحى أحمد على الشيخ . أستاذ الرياضيات البحتة بجامعة عين شمس الذي أحاطني بالرعاية والإهتمام من خلال اقتراحه لموضوعات البحث وإرشاده المستمر وصبره وعطائه اللامحدود وتوجيهه القيم. ولقد أعطاني الكثير من وقته الثمين. كما زودنى بحكمته والمعلومات العديدة من خلال السمينارات التى عقدت.

الدكتور/ رجب عبد القادر عمر . مدرس الرياضيات البحتة بجامعة عين شمس لدعمه المستمر وتشجيعه المتواصل ومناقشتة القيمة خلال إعداد هذه الرسالة.

كما أتقدم بالشكر الجزيل إلى قسم الرياضيات- كلية التربية - جامعة عين شمس ممثلاً برئيس القسم **الدكتور/ عثمان محمد فريج** الذي كان لنا خير مشجع وإلى أساتذتي وزملائي الكرام لما أحاطوني به من رعاية وإهتمام خلال سنوات الدراسة فلهم جزيل الشكر.

كما أتقدم بالشكر الجزيل إلى زملائي وأساتذتي رفقاء البحث العلمي في المدرسة التوبولوجية برئاسة **الأستاذ الدكتور/ علي قنديل سعد** الذي كان عوناً لي في إعداد هذه الرسالة ونوراً يضيء الظلمة التي كانت تقف أحياناً في طريق البحث العلمي والذي أخذ بيدي منذ الخطوة الأولى في مشوار البحث العلمي من خلال توجيهاته القيمة ودعمه المستمر وصبره وعطائه اللامحدود وجهوده المتواصلة خلال إعداد هذه الرسالة فجزاه الله عنا كل خير وله مناجزيل الشكر والإحترام والتقدير وأقول له بشراك قول رسول الله -صلى الله عليه وسلم -كما في سنن الترمذي " إن الله وملائكته وأهل السماوات والأرض، حتى النملة في جحرها وحتى الحوت، ليصلون على معلم الناس الخير "

كما أتقدم بالشكر **للدكتور/ محمد مصطفى ياقوت**،الذي ساعدني طيلة دراستي في هذا القسم.

وإلى أمي ،أبي وإخوتي الذين صبروا وتحملوا الشئ الكثير لأجلي فأنا مدين لهم بكل شئ.

Faculty of Education
Mathematics Department

Study some Modern Topological and Algebraic Structures

Submitted to:

The Department of Mathematics, Faculty of Education, Ain Shams University

A Thesis

Submitted in Partial Fulfillment of the Requirements of the Master's Degree in Teacher's Preparation in Science

(Pure Mathematics)

By

Mahmoud Raafat Mahmoud Soliman

Mathematics Demonstrator at,
Mathematics Department, Faculty of Education, Ain Shams University

Supervised by

Prof. Dr. S. A. El-Sheikh
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Acknowledgements

First of all gratitude and thanks to gracious **Allah** who always helps and guides me. I would like to thank **the prophet Mohamed** "peace be upon him" who urges us to seek knowledge and who is the teacher of mankind. This thesis would not be possible without the support of many individuals, to whom I would like to express my gratitude. First and foremost, I would like to thank my supervisors committee who are:

Prof. Dr. Sobhy Ahmed Aly El Sheikh, Professor of Pure Mathematics, Faculty of Education, Ain Shams University, who helped me at the first step in this study through his suggestions for the research problems, valuable instructions, guidance and continuous follow up in this study. He offered me much of his precious time and provided me with his wisdom and knowledge through many discussions we had. He learned me many things not only on the scientific side but also in practical and personal life. His efforts during revision of this thesis is an invaluable.

Dr. Ragab Abdelkader Omar, Lecturer of Pure Mathematics, Faculty of Education, Ain Shams University, who provided me with guidance and continuous encouragement. He did his best for the success of this work through seminars which were held, many discussions, precious comments, valuable reviews and remarks. His passion and extraordinary dedication to work have always inspired me and encouraged me to work harder.

Thanks also are due to **Dr. Osman Fereg**, Head of Mathematics Department, Faculty of Education, Ain Shams University, and all staff members for providing me with all facilities required to the success in this work.

I extend also my heartfelt thanks to scientific research fellows at the topological school headed by **Prof. Dr. Ali Kandil Saad**. who is the best commander for the march and that who helped me since the first step in the scientific research through his tremendous efforts, constant help and endless support, unlimited patience, infinite mercy, numerous discussions, valuable comments and remarks and his directives with all respect and appreciation.

In particular my great thanks to **Dr. Mohamed Mostafa Yakout**, who helped me at the first blush in this department, for all his effort.

Finally, I am appreciative to my kind parents and my beloved family for their support, patience, sacrifice and continuous encouragement. I owe my family everything.

Mahmoud Raafat

Contents

Summary

Multiset theory was introduced in 1986 by Yager [68]. A multiset is considered to be the generalization of a classical set. In classical set theory, a set is a well-defined collection of distinct objects. It states that a given element can appear only once in a set without repetition. So, the only possible relation between two mathematical objects is either they are equal or they are different. The situation in science and in ordinary life is not like this. If the repetitions of any object is allowed in a set, then a mathematical structure, that is known as multiset (mset [9] or bag [68], for short), is obtained in [11, 25, 59, 60]. For the sake of convenience an mset is written as $\{k_1/x_1, k_2/x_2, ..., k_n/x_n\}$ in which the element x_i occurs k_i times. The number of occurrences of an object x in an mset A, which is finite in most of the studies that involve msets, is called its multiplicity or characteristic value, usually denoted by $m_A(x)$ or $C_A(x)$ or simply by $A(x)$. Noted that each multiplicity k_i is a positive integer.

In Mathematics, the equation $x^2-4x+4=0$ has a solution $x=2,2$ which gives the multiset $S=\{2/2\}$. Additionally, One of the simplest examples is the multiset of prime factors of a positive integer n. The number 504 has the factorization $504=2^3 3^2 7^1$ which gives the mset $X=\{3/2, 2/3, 1/7\}$ where $C_X(2)=3$, $C_X(3)=2$, $C_X(7)=1$. In Chemistry, a water molecule H_2O is represented by the mset $M=\{2/H, 1/O\}$ and without one of the two hydrogen atoms, the water molecule is not created.

In the physical world it is observed that there is enormous repetition. For instance, there are many hydrogen atoms, many water molecules, many strands of DNA, etc. This leads to three possible relations between any two physical objects; they are different, they are the same but separate or they coincide and are identical. Many conclusive results were established by these authors and further study was carried on by Jena et al. [32] and many others [12, 13, 14, 47]. The notion of an mset is well established both in mathematics and computer science [9, 10, 15, 16, 23, 58, 61, 62].

A wide application of msets can be found in various branches of mathematics. Algebraic structures for mset space have been constructed by Ibrahim et al. [29]. In [53], Okamoto et al. used msets in coloring of graphs. Additionally, application of mset theory in decision making can be seen in [69]. In 2012, Girish and Sunil [26] introduced multiset topologies induced by mset relations. The same authors in [27] further studied the notions of open sets, closed sets, basis, sub-basis, closure, interior, continuity in multiset topological (M-topological, for short) spaces.

The concept of soft sets was first introduced by Molodtsov [48] in 1999 as a general mathematical tool for dealing with uncertain objects. In [48, 49], Molodtsov successfully applied the soft set theory in several directions, such as smoothness of functions, game theory, operations research, Riemann integration, Perron integration, probability, theory of measurement, and so on. In 2011, Shabir and Naz [56] initiated the study of soft topological spaces. They defined soft topology on the collection of soft sets over X. Consequently, they defined basic notions of soft topological spaces such as open soft sets and closed soft sets, soft subspace, soft closure, soft nbd of a point, soft separation axioms, soft regular spaces and soft normal spaces.

In 2013, Babitha et al. [8] and Tokat et al. [64] introduced the concept of soft mset (F, E) as $F : E \to PW(U)$ where E is a set of parameters and $PW(U)$ is a power whole mset of an mset U. Moreover, Tokat et al. [65] introduced the concept of soft mset (F, E) by another way as $F : E \to P^*(U)$ where E is a set of parameters and $P^*(U)$ is a power set of an mset U. The notion of a soft multiset in this thesis is the same as in [65, 66, 67]. In 2013, Tokat et al. [64] introduced the concept of soft multi topology and its basic properties. In addition, the notion of soft multi connectedness was studied in [65]. Additionally, the notion of soft multi compactness on soft multi topological spaces was presented in [66]. In 2015, Tokat et al. [67] presented the notion of soft multi continuous functions. The concept of soft msets which is combining soft sets and msets can be used to solve some real life problems. Also, this concept can be used in many areas, such as data storage, computer science, information science, medicine, engineering, etc.

A bitopological space (X, τ_1, τ_2) was introduced by Kelly [38] in 1963, as a method of generalizes topological spaces (X, τ). Every bitopological space (X, τ_1, τ_2) can be regarded as a topological space (X, τ) if $\tau_1 = \tau_2 = \tau$. Furthermore, he extended some of the standard results of separation axioms and mappings in a topological space to a bitopological space. The notion of connectedness in bitopological spaces has been studied by Pervin [54], Reily [55] and Swart [63].

In 1983 Mashhour et al. [44] introduced the notion of supra topological spaces by dropping only the intersection condition. Kandil et al. [33] generated a supra topological space (X, τ_{12}) from the bitopological space (X, τ_1, τ_2) and studied some properties of the space (X, τ_1, τ_2) via properties of the associated space (X, τ_{12}). Thereafter, a large number of papers have been written to generalize topological concepts to bitopological setting [17, 18, 19, 20, 34, 57].

Generalized open sets play a very important role in general topology and they are now the research topies of many topologists worldwide. Andrijevic [4, 7] introduced a class of generalized open sets in a topological space as b-open sets and α-sets. The class of b-open sets is contained in the class of β-open sets and contains both semi-open sets and pre-open sets. Levine [39], Mashhour et al. [43], Njastad [52] and Abd El-Monsef et al. [1] introduced semi-open sets, pre-open sets, α-open sets and β-open sets respectively.

This thesis is devoted to

1. Initiate supra M-topological spaces.
2. Study γ-operation and some types of msets in (supra) M-topological spaces.
3. Introduce generalized closed soft msets in soft multi topological spaces.
4. Study some types of soft msets and some types of soft multi continuous functions in soft multi topological spaces.
5. Introduce separation axioms, semi-compactness and semi-connectedness on (soft) multi topological spaces.
6. Given comparisons between the current results and the previous one by using counter examples.
7. Initiate multiset bitopological spaces and some operators on it.

This thesis contains 5 chapters:-

Chapter 1 is the introductory chapter, so it contains the basic concepts and properties of set theory and mset theory. It contains also the basic concepts and properties of topological space such as closure, interior, boundary, functions, separation axioms. The basic concepts and properties of bitopological spaces are presented. Further, this chapter contains the basic notions related to soft sets and soft topological spaces. Additionally, in Subsection 1.6, the concept of soft msets is introduced by Tokat et al. [65]. Moreover, the soft multi function and it's properties are introduced in this subsection.

In Chapter 2, the concept of supra M-topological spaces is introduced initially. Then, the notions of supra γ-operation, supra pre-open msets, supra α-open msets, supra semi open msets, supra β-open msets and supra b-open msets are presented. The properties of the present notions are studied and the relationships between them are given. The importance of this approach is that, the class of supra M-topological spaces is wider and more general than the class of M-topological spaces. For a special case, we introduced the notion of γ-operation in M-topological spaces.

Some results of this chapter are:

- “S. A. El-Sheikh, R. A-K. Omar and M. Raafat, γ-operation in M-topological space, *Gen. Math. Notes* 27 (2015) 40−54.”
- “S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Supra M-topological space and decompositions of some types of supra msets, *International Journal of Mathematics Trends and Technology* 20 (2015) 11−24.”

The goal of Chapter 3 is to study some (soft) multi topological properties in (soft) multi topological spaces which are represented by introducing separation axioms on M-topological spaces and study some of their properties. In addition, some algebraic structures on soft msets are obtained. Also, we introduced the notion of soft multi semi-compactness as a generalization of semi-compact in M-topological spaces and study its properties. Finally, we see that Theorem 1.6.3 in [65] is not correct and that is explained by a counter example. Moreover, the concept of semi-connectedness in soft multi topological spaces is introduced.

The results of this chapter are:

- “S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Separation axioms on multiset topological space, *Journal of New Theory* 7 (2015) 11−21.”
- “S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Semi-compact soft multi spaces, *Journal of New Theory* 6 (2015) 76−87.”
- “S. A. El-Sheikh, R. A-K. Omar and M. Raafat, A note on ”Connectedness on soft multi topological space”, *Journal of New Results in Science*, submitted.”

In Chapter 4, we introduced the concepts of generalized closed (open) soft msets and their properties. Also, the relationship between the current work and the previous one [39] is presented with the help of counter examples. Additionally, we introduced the concept of separated soft msets in soft multi topological spaces and study some results about this concept. The main purpose of Subsection 4.2 is to introduce the notions of γ-operation, pre-open soft msets, α-open msets, semi-open soft msets, β-open soft msets and b-open soft msets in soft multi topological spaces. The current notions are a generalization of the notions in [35]. In addition, the relationships among these types are studied. Moreover, the concepts of pre-continuous (respectively semi-continuous, α-continuous, β-continuous, b-continuous) soft multi functions are introduced and their properties are studied in detail. Also, the concepts of pre-irresolute (respectively semi-irresolute, α-irresolute, β-irresolute, b-irresolute) soft multi functions are presented.

Some results of this chapter are:

- "S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Some types of open soft multisets and some types of mappings in soft multi topological spaces, *Ann. Fuzzy Math. Inform.*, to appear."
- "S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Generalized closed soft multiset in soft multi topological spaces, *Asian Journal of Mathematics and Computer Research* 9 (2015) 302−311."

The main purpose of Chapter 5 is to introduce the notion of multiset bitopological spaces and study some M-operators on multiset bitopological spaces. Moreover, the notions of ij-operators such as ij-pre-open msets, ij-α-open msets, ij-semi-open msets and ij-β-open msets are presented on multiset bitopological spaces. The properties of these operators are studied and the relationships between them are given. Additionally, some deviations between M-topology and ordinary topology are given with the help of counter examples. The importance of this approach is that, the class of multiset bitopological spaces is more general than the class of bitopological spaces.

Some results of this chapter are:

- "S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Multiset bitopological spaces, *Asian Journal of Mathematics and Computer Research* 8 (2015) 103−115."
- "S. A. El-Sheikh, R. A-K. Omar and M. Raafat, Operators on multiset bitopological spaces, *South Asian J. Math.* 6 (2016) 1−9."

Chapter 1

Preliminaries

The purpose of this chapter is to present a short survey of some needed definitions and theories of the material used in this thesis.

1.1 Some basic concepts of topological structures

The aim of this section is to collect the relevant definitions and results from topology about interior, closure, boundary, separation axioms and mappings.

Definition 1.1.1 *[21] Let X be a nonempty set. A class τ of subsets of X is called a topology on X if it satisfies the following axioms:*

1. *$X, \phi \in \tau$,*
2. *an arbitrary union of the members of τ is in τ,*
3. *the intersection of any two sets in τ is in τ.*

The members of τ are then called τ-open sets, or simply open sets. The pair (X, τ) is called a topological space. A subset A of a topological space (X, τ) is called a closed set if its complement A^c is an open set. If τ satisfies the conditions 1 and 2 only, then τ is said to be a supra topology on X and the pair (X, τ) is called a supra topological space [44].

Definition 1.1.2 *[50] Let (X, τ) be a topological space and $A \subseteq X$. Then,*

1. *$cl(A) = \cap\{F \subseteq X : A \subseteq F$ and F is closed$\}$ is called the τ-closure of A,*