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GENERALIZED QUASILINEARIZATION FOR DEGENERATE SINGULAR PERTURBATION PROBLEMS

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To

Memory of My Father,

Memory of My Mother,

My Children, Aya, Alaa, Tasneem, Mohamad and Omar,

My Wife, and My Brother Mohamad.

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CHAPTER 1

INTRODUCTION

CHAPTER 1 INTRODUCTION

1.1 The Method of Upper and Lower solutions:

An interesting and fruitful technique for proving existence results for nonlinear problems is the method of upper and lower solutions. The basic idea is to modify the given problem suitably into a simpler problem, and then to employ known existence results of the modified problem, together with the theory of differential inequalities, to establish the existence results of the original problem. This method is sufficiently well known for boundary value problems whose study is substantially more difficult than that of initial value problems. Furthermore, the technique yields existence of solutions in a closed set, namely, the sector obtained by means of upper and lower solutions [8].

Generally speaking, the methods of proving existence results of nonlinear differential equations consist of three steps:

- (a) constructing a sequence of some kind of approximate solutions;
- (b) showing the convergence of the constructed sequence of approximate solutions; and
- (c) proving that the limit function is actually a solution of the given problem.

When we are dealing with continuous differential systems, steps (a) and (c) are standard and straightforward. It is step (b) that deserves attention and which leads, in turn, to three possibilities: it shows that the sequence of approximate solutions

- (1) is a Cauchy sequence in R,
- (2) satisfies the assumptions of the Ascoli-Arzela theorem [35], so that one can extract a uniformly convergent subsequence, or
- (3) is a monotone sequence that converges uniformly.