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TABLE OF CONTENTS

CHAPTER 1.

INTRODUCTION

1.1 Magnetohydrodynamics (MHD)

1.1.1 What is MHD?

MHD is concerned with the mutual interaction of fluid flow and magnetic field. The fluid in question must be electrically conducting and non-magnetic, which limits us to liquid metals, hot ionized gas and strong electrolytes. The mutual interaction of a magnetic field $\mathbf{H}$, and fluid velocity $\mathbf{u}$, arises partially as a result of the laws of Faraday and Ampere and because of the Lorentz force experienced by a current. It is convenient to split the process into three parts:

1- The relative movement of a conducting fluid and a magnetic field cause an electromotive force (e. m. f.) (of order $(\mathbf{u} \times \mathbf{B})$) is developed in accordance with Faraday's law of induction in general. Electric currents will be induced, with current density being of order $\sigma (\mathbf{u} \times \mathbf{B})$ where σ is the electrical conductivity.

2- These induced currents, according to Ampere's law, must give rise to a second, induced magnetic field. This will be added to the original magnetic field and the change is usually such that the fluid appears to drag the magnetic field lines along with it.

3- The combined magnetic field (imposed and induced) interacts with the induced current density, $\mathbf{J}$, to give rise to a Lorentz force, $(\mathbf{J} \times \mathbf{B})$. This acts on the conductor and is generally directed so as to inhibit the relative movement of the magnetic field and the fluid.

Note that these last two effects have similar consequence in both cases the relative movement of fluid and magnetic field tends to be reduced. Fluids can drag magnetic field lines and magnetic fields pull on conducting fluid. Let us consider the effect of a velocity, which influences an imposed magnetic field, depending on the product of the typical velocity of the motion, the conductivity of the fluid and a characteristic length scale. If the fluid is non-conducting or the velocity is negligible there will be no significant induced magnetic field. If the electrical conductivity or the velocity is large, then the induced magnetic field may alter the imposed field. The reason why the length is important is that the current density spread over a large area can produce a high magnetic field, whereas the same current density spread over a small area induces only a weak magnetic field. So that the product $\sigma \mathbf{u} l$ which determines the ratio of the induced field to the applied magnetic field have two limits. If this product tends to infinity the induced and the imposed magnetic fields are of the same order that occurs in the typical so called ideal conductor. In such cases it turns out that combined magnetic field behaves as if it were locked into the fluid. If it tends to zero the imposed magnetic field remains relatively unperturbed. Now the only difference between magnetohydrodynamic and conventional electrodynamic lies in the fluidity of the conductor. This makes the interaction between $\mathbf{u}$ and $\mathbf{B}$ more difficult to quantify [43].

1.1.2 Basic equations of MHD The basic equations of MHD consist of Maxwell's equations, continuity equation and momentum equation

1- Maxwell's equations:

$$\begin{aligned}
\nabla \cdot (\epsilon' \ \mathbf{E}) &= \rho_e \\
\nabla \times \mathbf{E} &= -\frac{\partial (\mu_e \ \mathbf{H})}{\partial t} \\
\nabla \cdot (\mu_e \ \mathbf{H}) &= 0 \\
\nabla \times \mathbf{H} &= \mathbf{J} + \frac{\partial (\epsilon' \mathbf{E})}{\partial t} \\
\frac{\partial \rho_e}{\partial t} + \nabla \cdot \mathbf{J} &= 0
\end{aligned} \tag{1.1}$$

where $\mathbf{H}$ is the magnetic field intensity, $\mathbf{E}$ is the electric field intensity, ρ_e is excess charge density, $\mathbf{J}$ is the total current density ϵ' and μ_e are constants (which depend on the material), called the electric permitivity and magnetic permeability, respectively, of the material.

2- The continuity equation:

$$\nabla \cdot \mathbf{u} = 0; \qquad \text{steady state} \tag{1.2}$$

where $\mathbf{u}$ is the fluid velocity.

3- The momentum equation:

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \, \mathbf{u} \right) = -\nabla \mathbf{p} + \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \mathbf{F}_e + \mathbf{F}_g \tag{1.3}$$

where $\mathbf{p}$ is the pressure, τ is the stress tensor, $\mathbf{F}_e$ is the electromagnetic force and $\mathbf{F}_g$ is the gravitational force.

Note that

$$\mathbf{B} = \mu_e\ \mathbf{H},$$

$$\mathbf{J} = I\ + \boldsymbol{\rho}_e \mathbf{u} = \boldsymbol{\sigma}\mathbf{E} + \boldsymbol{\rho}_e \mathbf{u},$$

$$\mathbf{F}_e = \boldsymbol{\rho}_\mathbf{e} \mathbf{E} + \mathbf{J} \times \mathbf{B}.$$

where $\mathbf{B}$ is the magnetic flux density, I is the electric current and σ is the electrical conductivity.

MHD equations are derived under the following assumptions, the motion of the fluid is non-relativistic that $\dfrac{|\mathbf{u}|}{c_\circ} << 1.$ the time scale is of the same order of magnitude of l where $\frac{l}{\mathbf{u}}$and $\mathbf{u}$ are the characteristic length and velocity respectively, that means the problem of very high frequency will not be considered. The electric field is assumed to be of the same order of magnitude of $(\mathbf{J} \times \mathbf{B})$. The electrostatic force due to charge density is negligible in comparison with magnetic force. The displacement current $\dfrac{\partial\left(\epsilon' \mathbf{E}\right)}{\partial t}$can be neglected and the term with excess electric charge $\boldsymbol{\rho}_e$ also negligible in comparison with other electromagnetic terms without $\boldsymbol{\rho}_e$. Under these assumptions we rewrite Maxwell's equation (1.1) as following

$$\nabla \cdot \mathbf{E} = 0,$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

$$\nabla \cdot \mathbf{B} = 0,$$

$$\nabla \times \mathbf{H} = \mathbf{J},$$

$$\nabla \cdot \mathbf{J} = 0. \tag{1.4}$$

from Eq. (1.4) we get

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) - \nabla \times (\nabla \times V_d \ \mathbf{H}) \tag{1.5}$$

Use the identity

$$\nabla \times (\nabla \times \mathbf{H}) = \nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H}$$

we get

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + \nabla^2 (V_d \ \mathbf{H}) \tag{1.6}$$

where $V_d = \dfrac{1}{\mu_e \sigma}$ is the magnetic diffustivity . If V_d is constant then Eq. (1.6) will be

$$\frac{\partial \mathbf{H}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{H}) + V_d \ \nabla^2 \mathbf{H} \tag{1.7}$$

The equation of motion under the MHD approximations will be

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \, \mathbf{u} \right) - \mu_e \, (\mathbf{H} \cdot \boldsymbol{\nabla}) \, \mathbf{H} = -\nabla \left(\mathbf{p} + \frac{1}{2} \mu_e \ \mathbf{H}^2 \right) + \boldsymbol{\nabla} \cdot \boldsymbol{\tau} + \mathbf{F}_g \tag{1.8}$$

Equations. (1.7), (1.8) are the fundamental equations of MHD; [166], [37] and [97].

1.2 Rayleigh-Taylor instability

The Rayleigh-Taylor instability is the instability of the plane interface between two fluids of different densities superposed one over the other accelerated towards each other;[29]. The nature of stability can be seen with a ball at the bottom of a valley contrasted to the unstable situation of a ball set on top of a hill, given a slight nudge in any direction, will continue in that direction going speed until it reaches the bottom of the hill. A ball at the bottom of valley will return to its initial position after a small displacement, however, the ball may very well oscillate about its initial position until the dissipative force of friction brings it to rest. In the context of the Rayleigh-Taylor instability, there are two fluids being considered, one above the other, which are separated by a horizontal plane, if the light fluid is above the heavy fluid, the interface between the two is stable because the heavy stuff may be considered to have already fallen and the light stuff has already risen. Stable does not imply there is no motion, if there is air above a fluid surface, a displacement of the surface will result in waves traveling horizontally on the surface. For a heavy fluid over a light fluid, the heavy stuff wants to fall and the light stuff wants to rise, however, for a perfectly flat interface is there are no avenues for this vertical motion to occur. When this perfectly flat interface is perturbed the waves generated provide the necessary means for the light fluid to rise in the peak of the wave and the heavy fluid to fall, in the valley of wave;[162]

The Rayleigh-Taylor instability also occurs when a light fluid is accelerated into a heavy fluid, and is a fundamental fluid-mixing mechanism. Understanding the rate of mixing caused by Rayleigh-Taylor instabilities, is important to a wide variety of application, including inertial confinement fusion, nuclear weapons explosions, stockpile managements and supernova explosions. In Rayleigh-Taylor instability the heavy fluid supported against gravity by a light fluid constitutes an unstable situation. Structures of light fluid penetrating the heavy are called bubbles, while the corresponding fingers of the heavy fluid are termed spikes as shown in Fig (1-1) when these modes are present, the interactions between modes result in a turbulent flow, characterized by a high level of mixing between fluids and self similarity. The mixing mechanism of

Rayleigh-Taylor instability magnified a small irregularities at the pusher fuel interface of thermonuclear which degrading its yield. Rayleigh-Taylor driven mixing also occurs in diverse applications ranging from supernova explosions to temperature inversions in the atmosphere;[130], [46] .

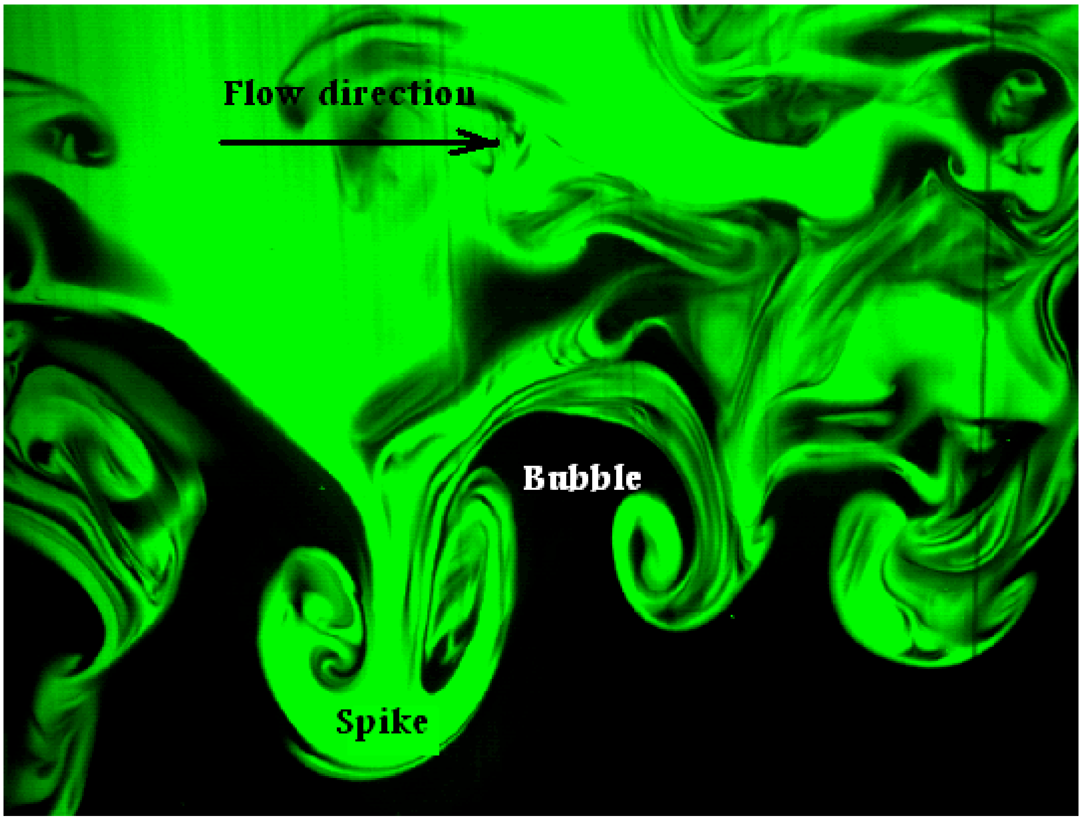

Fig. (1-1): Laser induced fluorescence image from a Rayleigh-Taylor experiment.

1.3 Gravitational Instability

A particularly simple example of gravitational instability was discovered by Jeans. In this example, we start from an infinite homogenous medium at rest and consider the velocity propagation of a small fluctuation in the density. If the gravitational effects of the fluctuation are ignored, the problem reduces to the classical one of the propagation of sound and as is well known, the velocity of sound c is independent of the wave number and is given by

$$c^2 = \frac{\gamma p}{\rho} \tag{1.9}$$

where $\gamma = \frac{c_p}{c_v}$, c_p and c_v are the specific heat under constant pressure and volume; respectively.

If the change in the gravitational potential consequent to the fluctuation in the density is taken into account, the velocity of wave propagation depends on the wavenumber k, and is, indeed, imaginary for all wavenumbers less than a certain value k_J. The instability which follows for $k < k_J$ is the gravitational instability discovered by Jeans.

The demonstration of the gravitational instability of an infinite homogenous medium is simple. The relevant perturbation equations are:

$$\rho \frac{\partial \mathbf{u}}{\partial t} = -\nabla \delta p + \rho \nabla \delta P \tag{1.10}$$

where ρ, δp, $\delta P, G$ are the density, pressure, gravitational potential and gravitational constant; respectively.

$$\frac{\partial \delta \rho}{\partial t} = -\rho \nabla . \mathbf{u} \tag{1.11}$$

$$\nabla^2 \delta P = -4\pi G \ \delta \rho \tag{1.12}$$

Further, we shall suppose that the fluctuations in the pressure and the density take place adiabatically so that