SOME PROBLEMS ON MAGNETO-HYDRO DYNAMICS WITH HEAT TRANSFER

THESIS

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By

Doaa Fouad Hussein Ezz El Arab
Special Diploma for Teacher Preparation in Science (Physics)
Faculty of Education – Ain Shams University(2001)

Under supervision of

Prof.Dr. Mohamed A. Kamel
Professor of Theoretical Physics,
Physics Department,
Faculty of Education,
Ain Shams University.

Prof.Dr. Elsayed F.El Shahawey
Professor of Applied Mathematics,
Mathematics Department,
Faculty of Education,
Ain Shams University.

Dr. Samia Elsayed Mustafa
Lecturer of Theoretical Physics,
Faculty of Education,
Physics Department,
Ain Shams University.

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Contents

SUMMARY

The main aim of the thesis is study numerically the velocity, heat transfer characteristics and the temperature profiles of the flow of a steady, incompressible and magneto fluid past a continuously moving plate in a porous medium in the presence of radiation. That is studied under the effect of variable thermal conductivity by using finite difference method. The thesis is consists three chapters.

The first chapter is a general introduction about fluids and their classification, magneto-hydrodynamic theory, heat transfer, modes of heat transfer and the boundary layer theory.

Also this chapter contains a summery of equations of motion and some of constitutive equations for different fluids.

A brief summery of magneto-hydrodynamic flow and heat transfer are found in this chapter .

The second chapter contains a study about magneto-hydro dynamic flow on a continuously moving porous plate in a porous medium in the presence of radiation .

By solving the set of the ordinary differential equations with boundary conditions using finite difference method with the help of a computational program. The effects of various parameters on the velocity and temperature have been studied. It has been found that;

1. The temperature increases with increasing magnetic parameter, relative temperature parameter.
2. The temperature decreases with increasing Reynolds number, Permeability parameter, Prandtle number and radiation parameter .
3. The velocity increases with increasing Reynolds number and Permeability parameter.
4. The velocity decreases with increasing Magnetic parameter.

The third chapter studies the Thermal-diffusion and diffusion-thermo effects in the presence of radiation and magnetic field on a mass transfer boundary layer flow through a porous medium with variable thermal conductivity. By solving the set of the ordinary differential equations with boundary conditions using finite difference method with the help of a computational program. We have studied the effects of various parameters on the velocity, temperature and concentration**.** It has been found that:

1. The velocity increases with the increase of the Permeability parameter, the mass buoyancy parameter, the Reynolds number and the temperature buoyancy parameter.
2. The velocity decreases with the increase of magnetic parameter, the radiation parameter and the thermal conductivity parameter.
3. The temperature increases with the increase of the magnetic parameter, the radiation parameter, the thermal conductivity parameter and the Dufour number.
4. The temperature decreases with the increase of the Permeability parameter, the temperature buoyancy parameter, the mass buoyancy parameter, the Reynolds number and the Prandtle number.
5. The concentration increases with the increase of the magnetic parameter, the thermal conductivity parameter and the Soret number.
6. The concentration decreases with the increase of radiation parameter, the Permeability parameter and the Schmidt number.

Appendices

APPENDIX 1

Finite difference method [61]

The idea of the finite difference representation of a derivative can be introduced by recalling the definition of derivative of the function F (x, y) at $x = x_o$, $y = y_o$

$$\frac{\partial F}{\partial x} = \lim_{\Delta x \to 0} \frac{F(x_0 + \Delta x, y_0) - F(x_0, y_0)}{\Delta x} \quad ,$$

A formal basis for developing finite difference approximation to derivatives is through the use of taylor series expansion. Consider taylor series expansion of a function F(x) about a point x, in the forward (positive x) and backward (negative x) directions given, respectively, by

(Forward)

$$F(x_0 + \Delta x) = F(x_0) + \left.\frac{df}{dx}\right|_0 \Delta x + \left.\frac{d^2 f}{dx^2}\right|_0 \frac{(\Delta x)^2}{2!} + \left.\frac{d^3 f}{dx^3}\right|_0 \frac{(\Delta x)^3}{3!} + \cdots\cdots\cdots,$$

(Backward)

$$F(x_0 - \Delta x) = F(x_0) - \left.\frac{df}{dx}\right|_0 \Delta x + \left.\frac{d^2 f}{dx^2}\right|_0 \frac{(\Delta x)^2}{2!} - \left.\frac{d^3 f}{dx^3}\right|_0 \frac{(\Delta x)^3}{3!} + \cdots\cdots\cdots,$$

These two expressions form the basis for developing finite difference approximations for first derivative df/dx about x_o. Rearranging the forward and backward equations we get the following expressions.

(Forward)

$$\left.\frac{dF}{dx}\right|_0 = \frac{F(x_0 + \Delta x) - F(x_0)}{\Delta x} + O(\Delta x) \quad ,$$

(Backward)

$$\left.\frac{dF}{dx}\right|_0 = \frac{F(x_0) - F(x_0 - \Delta x)}{\Delta x} + O(\Delta x) \quad ,$$

Where the order of notation "$O(\Delta x)$"characterizes the truncation error associated with the finite difference approximation. It represents the difference between the derivative and its finite difference representation.

$$O(\Delta x) \equiv \frac{\Delta x}{2} F''(x_0) + \frac{(\Delta x)^2}{6} F'''(x_0) + \cdots\cdots\cdots \quad ,$$

By subtracting the backward equation from the forward equation the central difference approximation is determined. The central difference approximation is a second order in (Δx), hence is a more accurate approximation than the forward and backward differences.

$$O(\Delta x)^2 \equiv \frac{(\Delta x)^2}{6} F'''(x_0) + \frac{(\Delta x)^5}{120} F'''''(x_0) + \cdots\cdots\cdots \quad ,$$

In the text below there are short notes about finite difference approximations for the first and second derivatives with two, three, and four point formulae.

Finite difference approximation of first derivative

Let n be the grid point at x_0. then the notation n+l and n-1 refer respectively, to the grid points at $(x_0 + \Delta x)$ and $(x_0 - \Delta x)$. Similarly the notation n+2 and n-2 refer, respectively, to the grid points at $(x_0 + 2\Delta x)$ and $(x_0 - 2\Delta x)$, and so on. We present below two, three, and four points formulae for the first derivative.

(Forward)

$$F_n^{'} = \frac{F_{n+1} - F_n}{h} + O(h) \quad ,$$

(Backward)

$$F_n^{'} = \frac{F_n - F_{n-1}}{h} + O(h) \quad ,$$

(Central)

$$F_n^{'} = \frac{F_{n+1} - F_{n-1}}{2h} + O(h^2) \quad ,$$

Where $h = \Delta x$.

The last form can be written as a general equation

$$F_n^{'} = \frac{(1-\varepsilon)F_{n+1} + 3\varepsilon\varepsilon_n - (1+\varepsilon)F_{n-1}}{2h} \quad ,$$

Where $\varepsilon = -1$ for forward, $\varepsilon = 0$ for central, $\varepsilon = 1$ for backward

$$O(h) = hF'' + \cdots\cdots \quad .$$

Finite difference approximation of second derivative

The Taylor series expansions given by the forward, and backward equations can be used to develop finite difference approximations for the second derivatives To obtain the central finite difference approximation for the second derivative forward and backward equations are added. The resulting expression is given as follow. (Central)

$$F_n^{''} = \frac{F_{n-1} - 2F_n + F_{n+1}}{h^2} + O(h^2) \, ,$$

Where

$$O(h^2) = \frac{h^2}{12} F_0^{''''} + \cdots\cdots\cdots ,$$

To develop forward and backward finite difference approximation for the second derivatives, the functions F$(x_0 + 2\Delta x)$ and F$(x_0 - 2\Delta x)$ are expanded in Taylor series. The function F'(x_0)is eliminated between the expansion of F$(x_0 + 2\Delta x)$and the

expansion F$(x_0 + \Delta x)$ and the resulting expression is solved for d^2F/dx^2 we get
(Forward)

$$F_n^{''} = \frac{F_n - 2F_{n+1} + F_{n+2}}{h^2} + O(h) \, ,$$

Similarly, the function F'(x_0)is eliminated between the expansion of F$(x_0 - 2\Delta x)$ and The expansion of F$(x_0 - \Delta x)$ and the resulting expression is solved for (d^2F/dx^2)
(Backward)

$$F_n^{''} = \frac{F_{n-2} - 2F_{n-1} + F_n}{h^2} + O(h) \, ,$$

Where

$$O(h) = hF_0^{'''} + \cdots\cdots\cdots \quad .$$

APPENDIX 2

Computer program to obtain the approximated solution for the equations of chapter II

```
□□ □10; n □25; h □ □□/n ;
Re □1; M □0.1; Kp □0.2; N □1; Pr □0.72; r □0.1; F_W □0.4;
Do FF1 i □ ExpandAll v i □ (f i □f i □1)/h , i, 0, n ;
Do FF2 i □ ExpandAll (v i □1 □2□v i □v i □1)/h^2 □
f i (v i □1 □v i □1)/(2□h) □ 2/RE □ MM□ 1/K v i , i, 0, n ;
Do FF3 i □ ExpandAll 3□NN□ (□ i □1 □2□ i □□ i □1)/h^2 □
□ □ i 3□ (□ i □1 □2□ i □□ i □1)/h^2 , i, 0, n ;
3□NN Pr f i (□ i □1 □□ i □1)/(2□h) □4□r^3□ 3 □ i 2□ (□ i □1 □□ i □1)/(2□h) ^2
f 0 □ F_W; v 0 □1; v n □0; □ 0 □1; □ n □0;
Timing $S□ FindRoot FF1 1 □ 0, FF1 2 □ 0, FF1 3 □ 0, FF1 4 □ 0
FF1 5 □ 0, FF1 6 □ 0, FF1 7 □ 0, FF1 8 □ 0, FF1 9 □ 0,
FF1 10 □ 0, FF1 11 □ 0, FF1 12 □ 0, FF1 13 □ 0, FF1 14 □ 0,
FF1 15 □ 0, FF1 16 □ 0, FF1 17 □ 0, FF1 18 □ 0, FF1 19 □ 0,
FF1 20 □ 0, FF1 21 □ 0, FF1 22 □ 0, FF1 23 □ 0, FF1 24 □ 0,
FF1 25 □ 0, FF2 1 □ 0, FF2 2 □ 0, FF2 3 □ 0, FF2 4 □ 0,
FF2 5 □ 0, FF2 6 □ 0, FF2 7 □ 0, FF2 8 □ 0, FF2 9 □ 0,
FF2 10 □ 0, FF2 11 □ 0, FF2 12 □ 0, FF2 13 □ 0, FF2 14 □ 0,
FF2 15 □ 0, FF2 16 □ 0, FF2 17 □ 0, FF2 18 □ 0, FF2 19 □ 0,
FF2 20 □ 0, FF2 21 □ 0, FF2 22 □ 0, FF2 23 □ 0, FF2 24 □ 0,
FF3 1 □ 0, FF3 2 □ 0, FF3 3 □ 0, FF3 4 □ 0, FF3 5 □ 0,
FF3 6 □ 0, FF3 7 □ 0, FF3 8 □ 0, FF3 9 □ 0, FF3 10 □ 0,
FF3 11 □ 0, FF3 12 □ 0, FF3 13 □ 0, FF3 14 □ 0, FF3 15 □ 0,
FF3 16 □ 0, FF3 17 □ 0, FF3 18 □ 0, FF3 19 □ 0, FF3 20 □ 0,
FF3 21 □ 0, FF3 22 □ 0, FF3 23 □ 0, FF3 24 □ 0 ,
f 1 , 0.027018 , f 2 , 0.1035 , f 3 , 0.21707 , f 4 , 0.3509 ,
f 5 , 0.48822 , f 6 , 0.61515 , f 7 , 0.72312 , f 8 , 0.80878 ,
f 9 , 0.87266 , f 10 , 0.91788 , f 11 , 0.948461 ,
f 12 , 0.96835 , f 13 , 0.9808 , f 14 ,
0.98854 , f 15 , 0.99314 , f 16 , 0.99586 ,
f 17 , 0.99586 , f 18 , 0.99586 , f 19 , 0.99586 ,
f 20 , 0.99586 , f 21 , 0.027018 , f 22 , 0.1035
```

```
⌊ 23 ⌊ 0.21707 ⌊ ⌊ 24 ⌊ 0.3509 ⌊ ⌊ 25 ⌊ 0.48822 ⌊
v 1 ⌊ 0 ⌊ v 2 ⌊ 0.005965 ⌊ v 3 ⌊ 0.0184919 ⌊ v 4 ⌊
0.069545 ⌊ v 5 ⌊ 0.0953029 ⌊ v 6 ⌊ 0.25749 ⌊ v 7 ⌊
0.16743 ⌊ v 8 ⌊ 0.265263 ⌊ v 9 ⌊ 0.039915 ⌊ v 10 ⌊
0.0242737 ⌊ v 11 ⌊ 0.0146551 ⌊ v 12 ⌊ 0.0086868 ⌊
v 13 ⌊ 0.001068 ⌊ v 14 ⌊ 0.0002586 ⌊ v 15 ⌊ 0.8 ⌊
v 16 ⌊ 0.81 ⌊ v 17 ⌊ 0.86 ⌊ v 18 ⌊ 0.88 ⌊ v 19 ⌊
0.89 ⌊ v 20 ⌊ 0.899 ⌊ v 21 ⌊ 0.91 ⌊ v 22 ⌊
0.95965 ⌊ v 23 ⌊ 0.994919 ⌊ v 24 ⌊ 1 ⌊
□ 1 ⌊ 1 ⌊ □ 2 ⌊ 0.95965 ⌊ □ 3 ⌊ 0.84919 ⌊ □ 4 ⌊
0.69545 ⌊ □ 5 ⌊ 0.53029 ⌊ □ 6 ⌊ 0.25749 ⌊ □ 7 ⌊
0.16743 ⌊ □ 8 ⌊ 0.065263 ⌊ □ 9 ⌊ 0.039915 ⌊ □ 10 ⌊
0.0242737 ⌊ □ 11 ⌊ 0.0146551 ⌊ □ 12 ⌊ 0.0086868 ⌊
□ 13 ⌊ 0.001068 ⌊ □ 14 ⌊ 0.0002586 ⌊ □ 15 ⌊ 0.8 ⌊
□ 16 ⌊ 0.66 ⌊ □ 17 ⌊ 0.47 ⌊ □ 18 ⌊ 0.698 ⌊ □ 19 ⌊
0.98 ⌊ □ 20 ⌊ 1 ⌊ □ 21 ⌊ 0.71 ⌊ □ 22 ⌊ 0.95965 ⌊
□ 23 ⌊ 0.84919 ⌊ □ 24 ⌊ 0.0 ⌊ MaxIterations □ 100 ⌊⌊
tv □ Table ⌊⌊N ⌊10 i / n ⌊ v ⌊ ⌊⌊ ss ⌊ ⌊ i, 0, n ⌊⌊
```

List Plot[tv,PlotJoined -> True,PlotRange->{0,1},
PlotStyle→{RGBColor[0,0,1]},AxesLabel→{η,f'}];

```
t□ □ Table ⌊⌊N ⌊10 i / n ⌊ □ ⌊ ⌊⌊ ss ⌊ ⌊ i, 0, n ⌊⌊
```

ListPlot[tθ,PlotJoined-> True,PlotStyle→{RGBColor
[1,0,0]},AxesLabel→{η,θ}];

```
Do ⌊PutAppend ⌊N ⌊□ i / n ⌊ "D:\m0.txt" ⌊ ⌊ i, 0, n ⌊
□□D:\m0.txt
Do ⌊PutAppend ⌊v ⌊ ⌊⌊ ss, "D:\mv.txt" ⌊ ⌊ i, 0, n ⌊
□□D:\mv.txt
Do ⌊PutAppend ⌊□ ⌊ ⌊⌊ ss, "D:\m1.txt" ⌊ ⌊ i, 0, n ⌊
□□D:\m1.txt
Evaluate ⌊ □11 v 0 ⌊⌊ ss □ 18 v 1 ⌊⌊ ss □ 9 v 2 ⌊⌊ ss □ 2 v 3 ⌊⌊ ss ⌊ / □6 h
Evaluate ⌊ □11 □ 0 ⌊⌊ ss □ 18 □ 1 ⌊⌊ ss □ 9 □ 2 ⌊⌊ ss □ 2 □ 3 ⌊⌊ ss ⌊ / □6 h
```

APPEDIX3

Computer program to obtain the approximated solution for the equations of chapter III

```
η∞ = 10; n = 25; h = η∞/n;
ggs = 0.1; ggc = 0.1; RE = 1; MM = 0.1; K = 0.2; Pr = 0.72;
Df = 0.1; r = 0.1; Sr = 0.1; Sc = 0.1; ss = 0.5;
Do[FF1[i] = ExpandAll[v[i] - ((f[i] - f[i - 1])/h)], {i, 0, n}];
Do[FF2[i] = ExpandAll[((v[i + 1] - 2 v[i] + v[i - 1])/h^2) +
 0.5 f[i] ((v[i + 1] - v[i - 1])/(2 h)) + ggs θ[i] + ggc HH[i]
 - (1/RE + MM + 1/K) v[i]], {i, 0, n}];
Do[FF3[i] = ExpandAll[(1 + ss θ[i]) ((θ[i + 1] - 2 θ[i] + θ[i - 1])/h^2) +
 ss ((θ[i + 1] - θ[i - 1])/(2 h))^2 + Pr (1/2 f[i] ((θ[i + 1] - θ[i - 1])/(2 h)) +
 Df ((HH[i + 1] - 2 HH[i] + HH[i - 1])/h^2) - r/RE θ[i])], {i, 0, n}];
Do[FF4[i] = ExpandAll[((HH[i + 1] - 2 HH[i] + HH[i - 1])/h^2) +
 Sc/2 f[i] ((HH[i + 1] - HH[i - 1])/(2 h)) + Sr Sc ((θ[i + 1] - 2 θ[i] + θ[i - 1])/h^2)], {i, 0, n}];
f[0] = 0; v[0] = 0; v[n] = 1; θ[0] = 1;
θ[n] = 0; HH[0] = 1; HH[n] = 0;
Timing[FindRoot[{FF1[1] == 0, FF1[2] == 0, FF1[3] == 0, FF1[4] == 0,
FF1[5] == 0, FF1[6] == 0, FF1[7] == 0, FF1[8] == 0, FF1[9] == 0,
FF1[10] == 0, FF1[11] == 0, FF1[12] == 0, FF1[13] == 0, FF1[14] == 0,
FF1[15] == 0, FF1[16] == 0, FF1[17] == 0, FF1[18] == 0, FF1[19] == 0,
FF1[20] == 0, FF1[21] == 0, FF1[22] == 0, FF1[23] == 0, FF1[24] == 0,
FF1[25] == 0, FF2[1] == 0, FF2[2] == 0, FF2[3] == 0, FF2[4] == 0,
FF2[5] == 0, FF2[6] == 0, FF2[7] == 0, FF2[8] == 0, FF2[9] == 0,
FF2[10] == 0, FF2[11] == 0, FF2[12] == 0, FF2[13] == 0, FF2[14] == 0,
FF2[15] == 0, FF2[16] == 0, FF2[17] == 0, FF2[18] == 0, FF2[19] == 0,
FF2[20] == 0, FF2[21] == 0, FF2[22] == 0, FF2[23] == 0, FF2[24] == 0,
FF3[1] == 0, FF3[2] == 0, FF3[3] == 0, FF3[4] == 0, FF3[5] == 0,
```