Generalizations of Fuzzy Ideals in Some Algebras

A THESIS

Submitted in Partial Fulfillment for the Requirement

Of

PhD. Degree for Teacher Preparation in Sciences (Pure Mathematics)

Abstract Algebra

To

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2013

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ACKNOWLEDGEMENTES

I wish to express my deepest gratitude and thankfulness to the late Professor Dr. Elsaid M. Elghazy, for his support and helpful suggestion during the preparation of this thesis.

With a great pleasure I wish to express my heartfelt sense of gratitude to my research supervisor, Associate Professor Dr. Mokhtar A. Abdel Naby for his invaluable suggestions, continuous encouragement, constant support, guidance and constructive criticism during the period of witting the thesis.

I would like to thank my supervisor, Associate Professor Dr. Samy M. Mostafa for suggesting the point of research and continuous guidance during the preparation of this thesis. Working with Dr. Samy was a truly pleasurable learning experience. I appreciate the time he spent working with me, and for teaching me how research is done.

Many thanks also to the chairman and staff of the department of mathematics, Faculty of Education, Ain Shams University, for their kind help and facilities offered through this investigation.

Contents

Abstract

In this thesis, we present new six notions of several classes of algebras. In order to make them clear, we firstly introduce the concept of interval-valued fuzzy Q-ideals in Q-algebras (briefly i-v fuzzy Q-ideals in Q-algebras). We study and obtain some related results. Secondly we present the notion of intuitionistic fuzzy Q-ideals in Q-algebras. We state and prove several theorems. Thirdly we introduce the notion of fuzzy TM-ideals in TM-algebras. Several theorems are stated and proved. Fourthly we provide the notion of interval-valued fuzzy KU-ideals in KU-algebras (briefly i-v fuzzy KU-ideals in KU-algebras). We study and investigate some of their properties. Fifthly we introduce the notion of intuitionistic fuzzy KU-ideals in KU-algebras. Several theorems are stated and proved. Finally we present the concept of fuzzy ideals in CI-algebras.

Key Words

Q-algebras; KU-algebras; TM-algebras; CI-algebras; TM-ideals; fuzzy TM-ideals; KU-ideals; fuzzy KU-ideals; Q-ideals; fuzzy Q-ideals; ideals in CI-algebras; fuzzy ideals in CI-algebras; interval-valued fuzzy Q-subalgebras; Interval-valued fuzzy Q-ideals; interval-valued fuzzy KU-subalgebras; interval-valued fuzzy KU-ideals; homomorphism of TM-algebra; intuitionistic fuzzy Q-ideal; intuitionistic fuzzy image (preimage) of Q-ideal; intuitionistic fuzzy KU-ideal; intuitionistic fuzzy image (preimage) of KU-ideal .

Introduction

In1966. Iséki and Imai [12, 13, 14] introduced two classes of algebras: BCK – algebra and BCI – algebra as a generalization of the concept of set-theoretic difference and prepositional calculus. It is now known that the class BCK – algebra is a proper subclass of the class of BCI – algebra. Since then, a great deal of literature has been produced on the theory of BCK / BCI – algebras. In particular, emphasis seems to have been put on the ideal theory of BCK / BCI – algebras. In [11], Hu and Li introduce a wide class of algebra : BCH – algebra. They showed that the class of BCI – algebra is a prober sub class of the class of BCH – algebra. Neggers, Ahn and Kim [37] introduced the notion of Q – algebra which is a generalization of BCK / BCI/ BCH – algebras. Moreover, Abdel Naby, Mostafa and Elgendy [1] introduced the notion of Q-ideal in Q-algebra. In [39], Prabpayak and Leerawat studied ideals and congruencies of BCC-algebras ([9], [10]) and they introduced a new algebraic structure which is KU-algebras. They gave the concept of homeomorphisms of KU – algebra and investigated some related properties. Recently, Meng [29] defined a CI-algebra which is related to BCK-algebra. In 2010, Megalai and Tamilarasi [28] introduced a class of abstract algebra: TM-algebra which is a generalization of Q / BCK / BCI / BCH-algebras.

The concept of fuzzy set and various operations on it were first introduced by Zadeh in [46]. Since then, fuzzy sets have been applied to various fields. The study of fuzzy sets and their application to mathematical contexts has reached to what is now commonly called fuzzy mathematics. Fuzzy algebra is an important branch of fuzzy mathematics. The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy group in 1971 by Rosenfeld [40], since then these ideas have been applied to other algebraic structures such as rings, ideals and vector spaces. In 1991, Xi [45] introduced the notion of fuzzy BCK – algebras. After that Jun and Meng [16] studied fuzzy BCK – algebra and fuzzy BCI – algebra. In 1993, Jun [18] defined fuzzy subsets in BCI – algebra and investigated some properties. In [47], Zadeh made an extension of the concept of fuzzy set by an interval valued set (i.e. a fuzzy set with an interval – valued membership). This interval – valued fuzzy set is referred to as an i-v fuzzy set. He constructed a method of approximate inference using his i-v fuzzy sets. The idea of intuitionistic fuzzy set was first published by Atanassov (see [3, 4]), as a generalization of the notion of fuzzy set. Fuzzy sets give a degree of membership of an element in a given set, while intuitionistic fuzzy sets give both degrees of membership and of non-membership. Both degrees belong to the interval [0, 1], and their sum should not exceed 1. Fuzzy and intuitionistic fuzzy set theories span a wide range of applications ranging from industrial process control to medical diagnosis and group decision processes. Original ideas and concepts in classical mathematical areas such as algebra, topology, and graph theory have been generalized with the rapid growth of both fuzzy and intuitionistic fuzzy set theories.

Summary of the thesis

The present thesis consists of seven chapters preceded by an introduction which includes a brief survey of algebraic structures that are available in literature.

Chapter one:

Gives, a comprehensive review of the published work for some several classes of algebras that is necessary for this thesis.

Chapter two:

In this chapter, we present the notion of interval-valued fuzzy Q-ideals (briefly i-v fuzzy Q-ideal) in Q-algebras. We study some of their properties. The image and the inverse image of i-v fuzzy Q-ideal are defined and we show how the homomorphic images and the inverse images of i-v fuzzy Q-ideal become i-v fuzzy Q-ideal in Q-algebra. Several theorems are stated and proved in this chapter.

Chapter three:

In this chapter we introduce the notion of intuitionistic fuzzy Q-ideals in Q-algebras. We study some of their properties. Also we introduce the notion of the product of intuitionistic fuzzy Q-ideals in Q-algebras and investigate some related properties. And then we study and investigate several properties which are related to this notion.

Chapter four:

Megalai and Tamilarasi [28] defined a new class of abstract algebra: TM-algebra. Then we introduce in this chapter a new notion which is TM-ideals and its fuzzification in TM-algebra. And then we study and investigate a lot of theorems and properties which are related to this notion.

Chapter five:

Prabpayak and Leerawat [39] introduced a new algebraic structured which is called KU – algebra and also Mostafa et al [32] introduced the notion of fuzzy KU – Ideal of KU – algebra . Then we present in this chapter a new notion which is interval-valued fuzzy KU-ideals (briefly i-v fuzzy KU-ideals) in KU-algebra. Several theorems are stated and proved in this chapter.

Chapter six:

In this chapter, we introduce the notion of intuitionistic fuzzy KU-ideals in KU-algebras. We study some of their properties, also we introduce the notion of the product of intuitionistic fuzzy KU-ideals in KU-algebras and we investigate some related properties to this notion.

Chapter seven:

Meng [29] defined a new class of abstract algebra: CI-algebra. Then we define an ideal and its fuzzification in CI-algebra to get a new algebraic notion which is fuzzy ideals in CI-algebra. Moreover, we study and state some of their properties.

Introduction

Chapter 1

PRELIMINARIES

§ 1-1 BCI / BCK-algebras

Definition 1.1.1[13]:

A BCI-algebra is non empty set X with a constant 0 and a binary operation $*$ satisfies the following axioms:

(I) $((x*y)*(x*z))*(z*y)=0$,

(II) $(x*(x*y))*y=0$,

(III) $x*x=0$,

(IV) $x*y=0$ and $y*x=0$ imply $x=y$, for all $x,y,z\in X$.

A BCI-algebra is said to be a BCK-algebra if it satisfies

(V) $0*x=0$, for all $x\in X$.

In a BCI-algebra X, a partially ordered relation $\leq$ can be defined by

$x\leq y$ if and only if $x*y=0$.

The following properties hold in every BCI / BCK-algebras (see [5, 6, 8, 14, 38, 43]):

(1) $(x*y)*z=(x*z)*y$,

(2) $x*0=x$,

(3) $0*(x*y)=(0*x)*(0*y)$,

(4) $0*(0*(x*y))=0*(y*x)$,

(5) $(x*z)*(y*z)\leq x*y$,

(6) $x*y=0$ implies $x*z\leq y*z$ and $z*y\leq z*x$, for all $x,y,z\in X$.

(7) A BCI-algebras X is called associative if $(x*y)*z=x*(y*z)$, for all $x,y,z\in X$.

(8) Let X be a BCI-algebra, for $x,y\in X$. x,y are said to be a comparable if $x\leq y$ or $y\leq x$.

Definition 1.1.2[30]:

A BCK-algebra $(X;*,0)$ is said to be

- Positive implicative if it satisfies for all x, y in X,

  $(x * z) * (y * z) = (x * z) * z$.

- Commutative if it satisfies for all x, y in X,

  $x * (x * y) = y * (y * x)$.

- Implicative if $x = x * (y * x)$, for all $x, y \in X$.

Example 1.1.3 [30]:

Let $X = \{0, 1, 2\}$ in which the operation $*$ is given by the table

$*$	0	1	2
0	0	0	0
1	1	0	1
2	2	2	0

Then $(X; *, 0)$ is an implicative BCK-algebra .

Note: the binary operation $*$ is defined as follow

$$\boxed{1} * \boxed{0} = 1$$