



Cairo University

A NEW FORMULATION FOR BI-OBJECTIVE OPTIMIZATION THROUGH A COMBINED APPROACH

By

Mohamed Shahat Abdel-Azim Badawi

A Thesis submitted to the
Faculty of Engineering at Cairo University

In Partial Fulfillment of the
Requirements for the Degree of

MASTER OF SCIENCE

In

**MECHANICAL DESIGN AND PRODUCTION
ENGINEERING**

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Key Words: Bi-objective Optimization, Feasibility Robustness, Objective Robustness, Pareto Solutions, Perturbation Analysis, Robustness Index.

Summary:

In this work, we present a new robust optimization approach based on the combined conventional and minimum sensitivity optimization approaches. For the new approach, we provide a new formulation in which the conventional objective function and the minimum sensitivity function are solved in a bi-objective optimization problem. A perturbation analysis is carried using Monte Carlo (MC) simulation approach. A comparison among the three optimization approaches is provided. The validity of the new approach is ascertained through test bed problems. Five case studies involving the optimization of real engineering systems are presented using the developed approach. An optimization code is developed using Matlab environment.

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DEDICATION

To my father.....

TABLE OF CONTENTS

ACKNOWLEDGEMENTS	i
DEDICATION	ii
TABLE OF CONTENTS	iii
LIST OF TABLES	vi
LIST OF FIGURES	vii
NOMENCLATURE	viii
ABSTRACT	x
CHAPTER 1: INTRODUCTION	1
1.1. MOTIVATION AND OBJECTIVE	1
1.2. RESEARCH COMPONENTS	2
1.2.1. Conventional Optimization vs. Objective Robustness	2
1.2.2. Feasibility Robustness Optimization	3
1.3. ORGANIZATION OF THESIS	3
CHAPTER 2: LITERATURE REVIEW	4
2.1. INTRODUCTION	4
2.2. EXPERIMENT-BASED ROBUST DESIGN METHODS	5
2.2.1. Simulation-Based Robust Design	5
2.2.2. Statistical Methods and Robust Modeling Techniques	5
2.2.3. The Robust Concept Exploration Method (RCEM)	7
2.3. MODEL-BASED ROBUST DESIGN METHODS	8
2.3.1. Stochastic Robust Design Techniques (SRDT)	8
2.3.2. Deterministic Robust Design Techniques (DRDT)	10
2.4. CONVENTIONAL vs. ROBUST OPTIMIZATION APPROACHES	12
2.4.1. Conventional Optimization Approach	12
2.4.2. Minimum Sensitivity Approach	13
2.5. MONTE CARLO (MC) SIMULATION	14
2.6. CRITICAL REVIEW OF PAST WORK	16
CHAPTER 3: THE PROPOSED COMBINED APPROACH	19
3.1. INTRODUCTION	19
3.2. THE COMBINED APPROACH	19

3.3. THE USED ALGORITHM	21
3.4. COMPARISON STUDY	23
3.5. INDEX OF ROBUSTNESS (β)	26
3.6. SUMMARY	28
CHAPTER 4: COMPARISON OF THE THREE APPROACHES	30
4.1. INTRODUCTION	30
4.2. TEST PROBLEMS	30
4.3. β – BASED STATISTICAL ANALYSIS	32
4.4. EFFECT OF SAMPLE SIZE	36
4.5. CONVENTIONAL vs. ROBUST SOLUTIONS	40
4.6. COMPARISON OF THE THREE APPROACHES WITH RESPECT TO ROBUST OPTIMUM	45
4.7. EFFECT OF WEIGHT GENERATION ON THE QUALITY OF SOLUTIONS	46
4.8. COMPARISON STUDY	47
4.9. THE MINIMUM SENSITIVITY APPROACH USING MULTI- OBJECTIVE OPTIMIZATION	48
4.10. CONCLUSIONS	50
CHAPTER 5: CASE STUDIES	51
5.1. INTRODUCTION	51
5.2. TWO-BAR TRUSS (RAO, 1996) - PROBLEM FORMULATION	51
5.3. TWO-BAR TRUSS (JUNG AND LEE, 2002) – PROBLEM FORMULATION	54
5.4. WELDED BEAM - PROBLEM FORMULATION	56
5.5. PRESSURE VESSEL - PROBLEM FORMULATION	59
5.6. METAL CUTTING - PROBLEM FORMULATION	62
5.7. CONCLUSION	64
CHAPTER 6: CONCLUSIONS AND FUTURE RESEARCH	65
REFERENCES	66
APPENDIX A: TEST SUITE PROBLEMS	72
APPENDIX B: SAMPLE OF THE DEVELOPED CODES	78
APPENDIX C: PLOTS OF PARETO FRONTIERS OF PROBLEMS 1-24	83

APPENDIX D: PERTURBATION RESULTS OF PROBLEMS 1-24	95
APPENDIX E: PERTURBATION RESULTS OF THE MINIMUM SENSITIVITY FORMULATION OF PROBLEM 11	99

LIST OF TABLES

Table 3.1: β values of the illustrative example	27
Table 4.1: Test problems used for validation	31
Table 4.2: β at the sample sizes 10,000 and 50,000	38
Table 4.3: β at the sample sizes 100,000 and 500,000	39
Table 4.4: Optimum solutions for the three approaches in problem 2	40
Table 4.5: Optimum solutions for the three approaches in problem 4	41
Table 4.6: Optimum solutions for the three approaches in problem 7	41
Table 4.7: Optimum solutions of the three approaches in problem 8	42
Table 4.8: Optimum solutions of the three approaches in problem 10	43
Table 4.9: Optimum solutions of the three approaches in problem 20	43
Table 4.10: Optimum solutions for problems including equality constraints	44
Table 4.11: Comparison among the three approaches	47
Table 4.12: Optimum solutions of the three formulations	49
Table 5.1: β of two-bar truss (Rao, 1996)	53
Table 5.2: Optimum solutions of two-bar truss (Rao, 1996)	53
Table 5.3: β of two-bar truss (Junge and Lee, 2002)	55
Table 5.4: Optimum solutions of two-bar truss (Junge and Lee, 2002)	55
Table 5.5: β of welded beam (Erfani and Utyuzhnikov, 2012)	58
Table 5.6: Optimum solutions of welded beam (Erfani and Utyuzhnikov, 2012)	58
Table 5.7: β of pressure vessel (Erfani and Utyuzhnikov, 2012)	61
Table 5.8: Optimum solutions of pressure vessel (Erfani and Utyuzhnikov, 2012)	61
Table 5.9: β of optimal machining (Ravindran et al., 2006)	63
Table 5.10: Optimum solutions of optimal machining (Ravindran et al., 2006)	63

LIST OF FIGURES

Figure 2.1: Difference between optimum and robust solutions (Lewis et al., 2001)	13
Figure 2.2: MC simulation of physical system	15
Figure 2.3: Some MC applications	16
Figure 2.4: Classification of robust design and optimization techniques	17
Figure 3.1: Proposed methodology	20
Figure 3.2: Pareto fronts for the illustrative example	26
Figure 3.3: Pareto solutions using the three approaches	27
Figure 3.4: Visualization of feasibility robustness	28
Figure 4.1: Contour plot – problem 2	33
Figure 4.2: Contour plot – problem 20	34
Figure 4.3: Contour plot – problem 11	34
Figure 4.4: Effect of sample size on β	37
Figure 4.5: Objective function vs. number of iterations (problem 1)	45
Figure 4.6: Objective function vs. number of iterations (problem 9)	46
Figure 4.7: Pareto frontier – minimum sensitivity formulation (problem 11)	49
Figure 5.1: Two-bar truss (Rao, 1996)	52
Figure 5.2: Pareto frontier of two-bar truss (Rao, 1996)	53
Figure 5.3: Two-bar truss (Junge and Lee, 2002)	54
Figure 5.4: Pareto frontier of two-bar truss (Junge and Lee, 2002)	56
Figure 5.5: Welded beam (Erfani and Utyuzhnikov, 2012)	57
Figure 5.6: Pareto frontier of welded beam (Erfani and Utyuzhnikov, 2012)	59
Figure 5.7: Pressure vessel (Erfani and Utyuzhnikov, 2012)	60
Figure 5.8: Pareto frontier of a pressure vessel (Erfani and Utyuzhnikov, 2012)	61
Figure 5.9: Pareto frontier of optimal machining (Ravindran et al., 2006).....	63

NOMENCLATURE

f	Response of a system
f_1	Main objective function
f_2	1 st derivative of the main objective w.r.t. to the design variables
F_1	Minimum sensitivity formulation 1
F_2	Minimum sensitivity formulation 2
F_3	Minimum sensitivity formulation 3
b	Vector of design variables
b^*	Values of design variables at minimum sensitivity
pdf's	Probability density functions
x	Uncertain variable
x^e	Estimated value of x
x_i	Design variable number i
$x_i \pm 1 \%$	Perturbation interval of 1 %
$x_i \pm 5 \%$	Perturbation interval of 5 %
$x_i \pm 10 \%$	Perturbation interval of 10 %
β	Robustness index
# of V	Number of variables
# of C	Number of constraints
N.O.C.	Nature of constraints
# of O.F _s .	Number of objective functions
Eq.	Equality
Ineq.	Inequality
CBFS	Correlation Based Feature Selection
CCD	Central Composite Design
CLT	Central Limit Theorem
DOE	Design of Experiments
DRDT	Deterministic Robust Design Techniques
DRSA	Dual Response Surface Approach
DVHS	Design Variation Hyper Sphere
FOSM	First Order Second Moment

GA	Genetic Algorithm
L	Linear
MC	Monte Carlo simulation
MPNN	Mathematical Programming Neural Network
MPP	Most Probable Point
MS	Minimum Sensitivity Approach
NL	Nonlinear
OA	Orthogonal Arrays
POSA	Post Optimality Sensitivity Analysis
PP	Physical Programming
RBDM	Robust Bayesian Data Mining
RCEM	Robust Concept Exploration Method
RDO	Robust Design Optimization
RMOGA	Robust Multi Objective Genetic Algorithm
RPD	Robust Parameter Design
RS	Response Surfaces
RSM	Response Surface Methodology
SI	Sensitivity Index
SPSA	Simultaneous Perturbation Stochastic Approximation
SQP	Sequential Quadratic Programming
SRDT	Stochastic Robust Design Techniques
TRF	Tunable Robust Function

ABSTRACT

Developing robust design solutions with respect to performance, as well as feasibility, is one of the most important concerns in engineering optimization. In this work, a new robust optimization approach is presented. It incorporates the two main concepts of robust optimization: objective robustness and feasibility robustness. The majority of robust optimization techniques consider one of the two concepts of robustness. The new approach is based on the combined conventional and minimum sensitivity optimization approaches. The conventional optimization approach provides a solution which has the highest performance, where the minimum sensitivity approach develops a solution with the least sensitivity to variations. For the new approach, a new formulation is provided, in which the conventional objective function and the minimum sensitivity function are solved in a bi-objective optimization problem. A perturbation analysis is carried using Monte Carlo (MC) simulation approach. A comparison among the three optimization approaches (conventional, minimum sensitivity, and combined) is provided. The validity of the new approach is ascertained through test bed problems. Five case studies involving the optimization of real engineering systems are presented using the developed approach. An optimization code is developed using Matlab environment.

CHAPTER 1

INTRODUCTION

1.1. MOTIVATION AND OBJECTIVE

Uncontrollable variations and noises are unavoidable in engineering design. Temperature variations, deviations of material properties from specifications, and dimensional tolerances of a design are just few examples of uncontrollable parameter variations. When designing a system, these variations cannot be ignored because they can seriously affect the performance of a design. One way to counter the effects of these variations is to try to reduce or eliminate the parameter variations themselves. However, this approach is usually very difficult to undertake and/or expensive to implement. Furthermore, it is quite possible that such variations will re-appear. A better approach is to try to reduce the sensitivity of the design to variations so that deteriorations caused by these variations are kept within an acceptable level.

Genichi Taguchi (Taguchi, 1993) introduced the idea of reducing the sensitivity of a design, through parameter design. Since then, this "*least-sensitive design*" idea is developed and the "*robust design*" is coined. Later, "*objective robustness*" and "*feasibility robustness*" are introduced to refer to the robustness with respect to the objective and constraint functions in an optimization problem.

The first robust optimization approach is advocated by Belegundu and Zhang (1989). This method is called the minimum sensitivity approach. It assumes the existence of analytical model and attempts to minimize the sensitivity with respect to decision variables through minimization of the first derivatives with respect to design parameters. This approach is straightforward and problem-independent manner. No assumptions are made on the probability distributions of uncertain variables. Compared to the conventional deterministic optimization, the minimum sensitivity approach provides a robust design that is least sensitive to variations. However, the existence of the solution close to one or more constraint results in infeasible solution.

It is important to achieve robust design objectives, but also to maintain the robustness of design feasibility under the effect of variations. The evaluation of feasibility robustness is often a computationally extensive process. Simplified