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FOLDING OF STAR MANIFOLDS

A THESIS

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By

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الله

بسم الله الرحمن الرحيم

قالوا سبحانك لا علم لنا إلا ما
علمتنا إنك أنت العليم الحكيم .

صدق الله العظيم
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ABSTRACT

The concept of isometric foldings of Riemannian manifolds can be extended to a much wider class of manifolds without lossing the main local structure theorem, we present here the folding concept of star manifolds.

The notation of star folding of star manifolds is based on 1-spreads, where the rule of geodesics on Riemannian manifolds is assumed by smooth, unoriented and unparameterised curves on smooth manifolds.

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CHAPTER ONE

RIEMANNIAN MANIFOLDS

The concept of smooth manifolds began to appear in vague forms about a hundered years ago , when in several branches of mathematics the need arose to study topological spaces whose structure locally was euclidean (real or complex) but which was globally certainly not euclidean , that is topological spaces that locally " look the same " as some euclidean spaces .

The most familiar examples of manifolds are smooth surfaces like sphere or tours (doughnut) , where each point lies in a little curved disk that may be gently fllattened into a disk in the plane .

see Fig. (1-1)

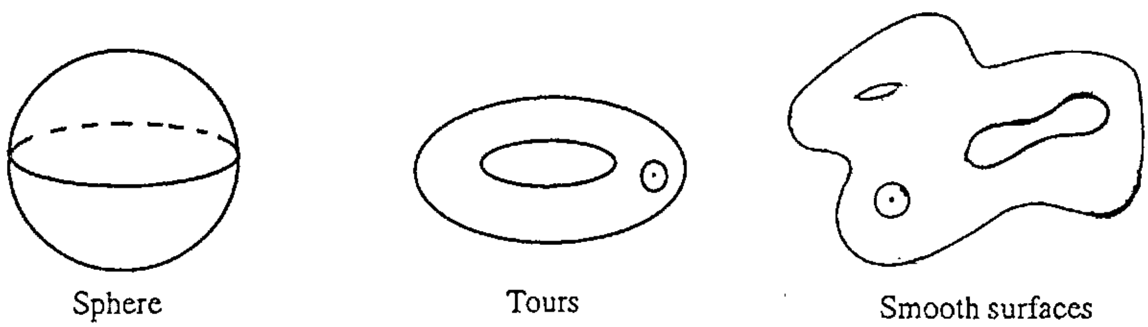
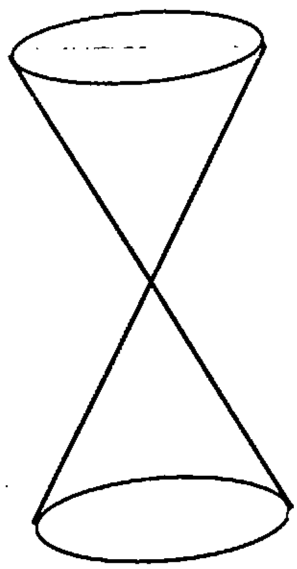


Figure (1 - 1)

A surface which does not qualify as a manifold is the cone . Every point but one has a nice euclidean environment , but no neighbourhood of the vertex point looks like a simple piece of the plane, see Fig. (1-2)



Cone

Figure (1-2)

To translate this idea into mathematical definition , we need to make precise the criterion " sameness " . This can be done in terms of mapping as we shall see in the following .

(1-1) Differentiable functions and diffeomorphism on euclidean spaces

A function $f : R^n \to R$ is said to be class C^∞ at a point x of its domain if every partial derivative of f exists and is continuous on some neighbourhood of x .

A function $f : R^n \to R^m$ is said to be class C^∞ at a point x if the real-valued functions $g_i = p_i \circ f$, $i = 1 , \ldots , m$, $p_i : R^m \to R$ are all class C^∞ at x . This function is smooth (C^∞ - function) if it is class C^∞ at each point of its domain . We notice that the domain of such a function must be an open subset of R^n .

A map $f : X \rightarrow R^m$ defined on aribitrary subset of X in R^n is called smooth if it may be locally extended to a smooth map $F: U \rightarrow R^m$ such

that F equals f on $U \cap X$, [13]. See Fig. (1-3) .

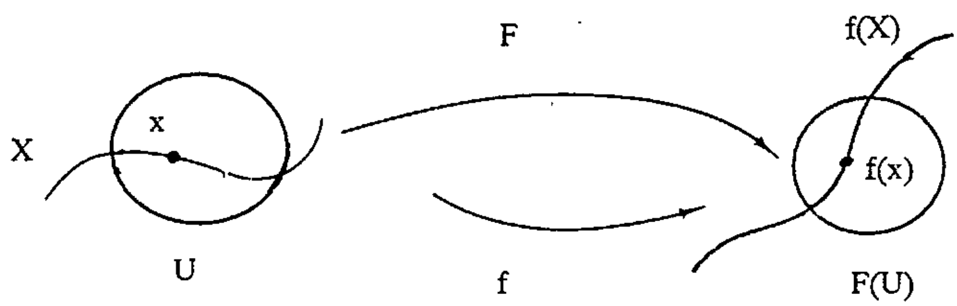


Figure (1-3)

A smooth map $f : X \rightarrow Y$ of subsets of two euclidean spaces is a ***diffeomorphism*** if it is one to one , onto , and the inverse map f^{-1} : $Y \rightarrow X$ is also smooth . If such a map exists the two spaces X and Y are said to be ***diffeomorphic*** i.e. a homeomorphism f is a diffeomophism if and only if f and f^{-1} are both smooth. It should be noted that a smooth homeomorphism need not be a diffeomorphism , e.g. $f:R \rightarrow R$ (with usual structures) given by $h(x) = x^3$.

Fig. (1-4) gives us some examples of diffeomorphic and not diffeomorphic spaces

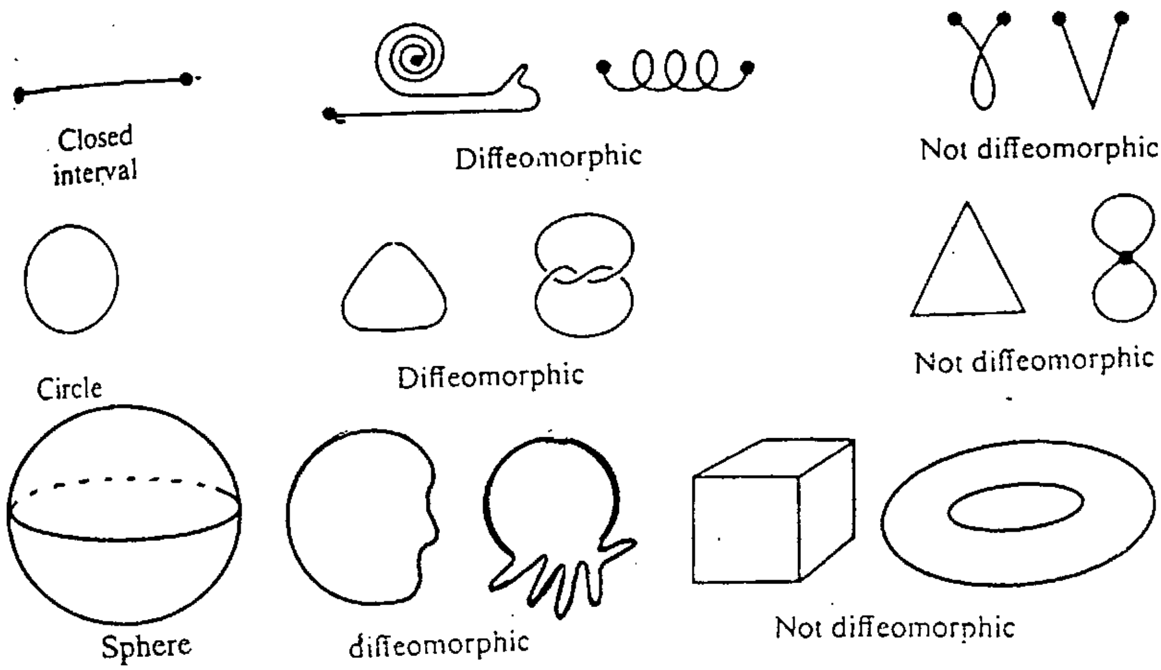


Figure (1-4)

In the following we will give the precise mathematical definition of smooth manifolds with some examples to explore the definition .

(1-2) Smooth manifolds :

Let M be non-empty (second-countable) Hausdorff topological space such that :

(i) M is a union of open subsets U_α, and each U_α is equipped with a homeomorphism ϕ_α taking U_α to an open set in R^n , i.e,

$$\phi_\alpha : U_\alpha \to \phi_\alpha (U_\alpha) \subset R^n$$

(ii) If $U_\alpha \cap U_\beta \neq \phi$, then the overlap map

$$\phi_\beta \phi_\alpha^{-1} : \phi_\alpha(U_\alpha \cap U_\beta) \to \phi_\beta (U_\alpha \cap U_\beta)$$