# SYMBOLIC-BASED REPRESENTATION AND ANALYSIS OF PARAMETER VARYING SYSTEMS

**By**
**Eng. Mohamed Saleh El Sayed Ahmed**

A Thesis Submitted to the
Faculty of Engineering at Cairo University
In Partial Fulfillment of the
Requirements for the Degree of
**DOCTOR OF PHILOSOPHY**
In
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**Title of Thesis:**

SYMBOLIC-BASED REPRESENTATION AND ANALYSIS OF PARAMETER VARYING SYSTEMS

**Key Words:**
Generic control system design; Parameters varying systems; Embedded symbolic function systems; Continuous (infinite) modal system design; Symbolic computations.

**Summary:**

This thesis addresses the problem of exact generic symbolic formulation, derivation and solution Parameters Varying Systems (PVS) problems in control systems. Comparison between the exact generic symbolic mathematical approaches versus various techniques is investigated. The comparisons revealed that the generality and flexibility of the exact generic symbolic mathematical concept. The parameter varying systems are addressed from the physical, mathematical, and representation perspectives. The proof of concept of the new suggested concept is shown by solving different applications. For control applications, the notion of symbolic VPS control strategy realization is carried out through the incorporation of embedded configurable function units. Finally, such presented notions in this research represent a new leap towards moving back to the roots of exact generic symbolic systems.

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I would finally ask ***Allah*** that this present study contributes in the nourishment of science and serving humanity as well.

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# List of abbreviations

| | |
|---|---|
| ASICs | Application-specific Integrated Circuits |
| CISC | complex instruction set computer |
| ECSs | Embedded Computing Systems |
| FP | Functional Programming |
| FPGAs | Field-programmable gate arrays |
| GSCM | generic symbolic continuous modal |
| IMU | Inertial measurement unit |
| INS | Inertial navigation system |
| LCD | Liquid Crystal Display |
| LFT | Linear fractional transformation |
| LPV | linear parameter varying |
| LTI | linear time-invariant |
| LTV | linear time-varying |
| MIMO | multiple input multiple output system |
| $M^{\infty}$ | infinite modal operation |
| PID | Proportional integral derivative |
| RISC | reduced instruction set computer |
| SISO | single input single output system |

# List of symbols

| | |
|---|---|
| $A_{aero}$ | Aerodynamic force. |
| $a$ | The absolute acceleration vector. |
| $a_{carr}$ | Carried acceleration. |
| $a_{cor}$ | Coriolis acceleration. |
| $a_g$ | Relative acceleration. |
| $C$ | Ballistic coefficient. |
| $C_D$ | Drag coefficient. |
| $C_L$ | Lift coefficient. |
| $F_T$ | Total resultant force. |
| $F_{Refs}$ | Merged features matrix. |
| $F_{Refs\_SURF}$ | SURF features matrix. |
| $F_{Refs\_MSER}$ | MSER features matrix. |
| $g$ | Gravity acceleration [m/s$^2$] |
| $H_1$ | Body angular momentum vector. |
| $h$ | Vehicle altitude. |
| $I$ | Moment of inertia of the body. |
| $J_2$ | Constant used to compute the gravitational acceleration. |
| $M$ | Total aerodynamic moment. |
| $m$ | Vehicle mass. |
| $P_{thr}$ | Thrust force. |
| $q$ | Dynamic pressure. |
| $\Delta_\rho$ | a set that contains parameters values |
| $R_e$ | Equatorial radius of the earth. |
| S | Reference area of flying body. |
| s | Sin |
| c | Cos |
| $V_a$ | Vehicle velocity vector. |
| $V_b$ | Vehicle velocity in body frame. |
| $V_g$ | Relative velocity. |
| $V_w$ | Wind velocity vector. |
| $w$ | Angular velocity. |
| $w_e$ | Earth angular velocity. |
| $w_r$ | Relative angular velocity. |
| $X$ | Drag force. |
| $Y$ | Lift force. |
| $Z$ | Side force. |
| $\lambda$ | Geodetic longitude. |

| | |
|---|---|
| $\phi$ | geodetic latitude. |
| $\alpha$ | Angle of attack. |
| $\beta$ | Side slip angle. |
| $\vartheta$ | Pitch angle. |
| $\psi$ | Yaw angle. |
| $\gamma$ | Roll angle. |
| $\rho$ | Air density. |
| $c_{xo}, c_x^{\alpha}, c_x^{\beta}$ | Coefficient of drag force, inductive resistance coefficient. |
| $c_y^{\alpha}, c_z^{\beta}$ | Lift force coefficient and lateral force coefficient respectively |
| $c_{x_{oru}}$ | drag coefficient of actuator |
| $c_{x_{ru}}^{\delta^2}$ | induced drag coefficient of actuator |
| $s_{ru}$ | area of actuator |
| $q_{ru}$ | dynamic pressure actuator |
| $\delta_1, \delta_2, \delta_3, \delta_4$ | deviation of actuators |

# Abstract

This thesis consists of a study of Parameter Varying Systems (PVS). New classifications of PVS symbolic derivations mechanisms and solution methods, as well as the operations symbolic coding are proposed. The PVS are addressed from the physical, mathematical, and representation perspectives. For expediting the productivity of the derivation of the symbolic (or algebraic) solutions, manual, interactive symbolic derivations manipulation or a combination of them are considered. In this way, fully programming (one-shot program) symbolic computation is achieved.

A new generic symbolic continuous modal (GSCM) approach for formulation and representation of different systems in different areas is proposed. In this approach, the system takes varying parameters form. The parameters are determined by sensing and feeding back of the parameters variations in a continuous manner using symbolic-based expressions in the corresponding control functions. The GSCM approach permits the simultaneous manipulation of system control and operation with the online changes of system parameters. The GSCM approach enables high system operation flexibility, while the continuous (infinite) modal operation ensures complete smoothing system behavior and full operation modal compatibility versus system varying parameters. The realization of the continuous (infinite) modal design is carried out through symbolic-based embedded control expressions using computational mathematics.

The proof of concept of proposed approach is demonstrated through two illustrative examples as well as an application representing the experimentation of control of inverted pendulum system with varying cart mass operation. The results of the implementation to the selected application are depicted comprising the executed embedded symbolic-based functions using microcontroller/Proteus simulation program configuration equipped with functional programming facility for feedback gains expressions executions.

Finally, it is recommended that shift should gradually turn towards extra exploration of the experimentation of symbolic-based embedded stabilization expressions within the framework of generic mathematical control strategies.

# Chapter 1
# Introduction

The control system is a set of components that act together and perform a certain function that has one or more well-defined inputs and one or more well-defined outputs. In studying control systems, the systems dynamics must be presented in mathematical formulation terms and an analysis of their dynamic characteristics is exerted. A system is called linear if the principle of superposition applies. Dynamic systems that are composed of linear time-invariant parameter components may be described by linear time-invariant differential equations. Such systems are called linear time-invariant (LTI) or linear constant-coefficient systems. Systems that are represented by differential equations whose coefficients are functions of time are called linear time-varying (LTV) systems. [1]

A system is described as linear parameter varying system (LPV) when its parameters are dependent on the varying signal p while the dynamic relation between input signals u and output signals y is still linear as shown in Figure 1.1. [2]

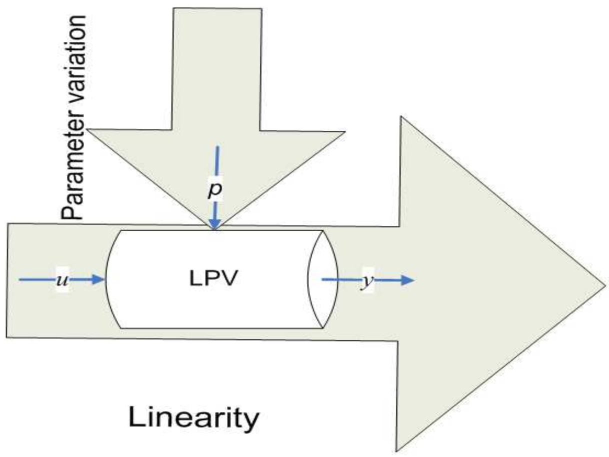


**Figure 1.1: Input-output signal flow of LPV systems.**

Linear parameter-varying (LPV) systems are linear dynamical systems whose mathematical description or state-space representations depends on parameters that change values over time.

These parameters are generally considered as bounded and taking values inside a set $\Delta_\rho$. LPV systems are commonly described by equations of the form

$$\begin{aligned} \dot{x}(t) &= A(\rho(t))x(t) + E(\rho(t))w(t),\ t \geq 0 \\ z(t) &= C(\rho(t))x(t) + F(\rho(t))w(t) \\ x(0) &= x_0 \end{aligned} \tag{1.1}$$

where $x$, $w$ and $z$ are the state, the input and the output of the system, respectively. The parameter vector $\rho$ acts internally on the system by modifying its structure overtime, and,