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A Graph Theory Approach for Optimizing the Circulation Problem in Networks

By

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Certification

I certify that this work has not been accepted in substance for any academic degree and is not being concurrently submitted in candidature for any other degree.
Any portions of this thesis for which is indebted to other sources are mentioned and explicit references are given.

Hanan Hussein

Summary

A Graph Theory Approach for Optimizing the Circulation Problem in Networks

Abstract

In this thesis we used an approach is suggested to define and apply graph theory which is different form of the classical one in the literature, this approach is used to illustrate and represent different circulation problems in network.

The thesis will show that the new definition is more appropriate to put the network circulation problem in more rigors formal expression which will make a computer based technique to solve this problem more adequate.

These days, graph theory is one of the most popular and fertile branches in mathematics and computer science. One important reason for this renewed interest in graph theory is its applicability to many of the complex and wide-ranging problems of modern society in such diverse fields as economics, facility location, management science, marketing, energy modeling, transmission of information, and transportation planning to name a few. Quite often such problems can be modeled as a graph or network. In this context graph theory is used first and foremost as a tool for formulating problems and defining structural inter relationships. Once a problem is formulated in graph-theoretical language, it becomes relatively easy to comprehend it in its generality. The next step will, of course, be to exploring avenues to seek a solution to the problem. The field of graph theory has two different braches: the algebraic aspects & the optimization aspects (the area of network optimization, which is greatly advanced by the advent of the computer).

The thesis has been organized in six sections. The first is introduction, the second is the new definition of graph theory, the third is the directed graph, the fourth flow in network, the five circulation problems the sixth applications with reseal directions.

Keywords (graph theory, network, flow, circulation, optimization, short path, assignment)

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List of symbols

G	A graph
V	Set of vertices
E	Set of edges
$V \cap E$	Intersection of the sets V and E
$\varnothing$	Empty set
ψ	The incidence function
$\psi : E \rightarrow \zeta$	ψ Is a function from E to ζ
$\psi(e)$	Image of e under the function ψ
ζ	Subset of non-Cartesian product
$V \times V$	Cartesian product of v
$V \overset{\bullet}{\times} V$	non-Cartesian product
$\lvert V \rvert$	Cardinality of V
$M(G)$	The incidence matrix of graph G
$A(G)$	The adjacent matrix of graph G
m_{ij}	Matrix with entries m_{ij}
a_{ij}	Matrix with entries a_{ij}
$\sum_{i=1}^{n} a_i$	$a_1 + a_2 + \dots\dots\dots\dots a_n$
$\prod_{i=1}^{n} a_i$	$a_1 a_2 a_3 \dots\dots\dots a_n$.
R	The field of real numbers
R^+	Set of positive real numbers
$\dot{M}D$	Main diagonal of non-Cartesian product
B	The partition of A
d (v)	The degrees of a vertex v
d_{eg+} (v),indeg	Indegree of vertex v
d_{eg-} (v),outdeg	Outdegree of vertex v
$\chi(G)$	Chromatic number of G
$\Delta(G)$	Maximal degree of a vertex of G

$\chi'(G)$	Chromatic index of G
W_{az}	The walk from a to z
W_{az}^{-1}	The walk from z to a
$\overrightarrow{W}_{az}$	Directed walk from a to z
T_{az}	The trail from a to z
P_{az}	The path from a to z
P_{az}^{-1}	The path from z to a
$\overrightarrow{P}_{az}$	Directed walk from a to z
C_{aa}	cycle
D	Directed graph
N	Set of nodes
A	Set of arcs
χ	function from Subset of Cartesian product of $N \times N$
S_{in}	Set of sink node
S_{out}	Set of source node
f	Flow in network
f^*	The maximum flow in network
b(a)	The lower capacity constraint for the arc a
c(a)	The upper capacity constraint for the arc a
$c(S_{out}, S_{in})$	Capacity of cut (S_{out}, S_{in})
$f(S_{out}, S_{in})$	Flow value for cut (S_{out}, S_{in}) with respect to the flow f
p	Number of connected components of D
$\gamma(a)$	The cost of arc a
$O(f(n))$	The upper bound on the complexity
$\Omega(f(n))$	The lower bound on the complexity
$\Theta(f(n))$	Rate of growth
iff	If and only if
$\exists$	There is exist

Chapter1

Introduction and Review

1-Introduction

In this thesis we used an approach is suggested to define and apply graph theory which is different form of the classical one in the literature, this approach is used to illustrate and represent different circulation problems in network.

The thesis will show that the new definition is more appropriate to put the network circulation problem in more rigors formal expression which will make a computer based technique to solve this problem more adequate.

These days, graph theory is one of the most popular and fertile branches in mathematics and computer science. One important reason for this renewed interest in graph theory is its applicability to many of the complex and wide-ranging problems of modern society in such diverse fields as economics, facility location, management science, marketing, energy modeling, transmission of information, and transportation planning to name a few. Quite often such problems can be modeled as a graph or network. In this context graph theory is used first and foremost as a tool for formulating problems and defining structural inter relationships. Once a problem is formulated in graph-theoretical language, it becomes relatively easy to comprehend it in its generality. The next step will, of course, be to exploring avenues to seek a solution to the problem. The field of graph theory has two different braches: the algebraic aspects & the optimization aspects (the area of network optimization, which is greatly advanced by the advent of the computer).

Problem definition

To define the graph and its importance, let us enumerate the following questions which will be answered in this thesis to give us a starting point for developing an approach for renewing the definition of the graph terminologies.

1- How can we lay cable at minimum cost to make every. Telephone reachable from every other?

2- What is the fastest route from the national capital to each state capital?

3- How can n jobs be filled by m people with maximum total utility?

4- What is the maximum flow parquet time from source to sink in network of pipes?

5- How many layers does a computer chip need so that wires in the same layer don't cross?

6- How can the season of a sports league be scheduled into the minimum number of weeks?

7- In what order should a traveling salesman visit cities to minimize travel time?

8- Can we color the regions of every map using four colors so that neigh boring regions receive different colors?

when the above questions are answered the questions above we shall deal with some application on operations research[33].

The thesis has been organized in six sections. The first is introduction, the second is the new definition of graph theory, the third is the directed graph, the fourth flow in network, the five circulation problems the sixth applications with reseal directions.

2-Review

Here the classical definitions of the main items in graph theory. These classical definitions are very popular and used almost all over, later in our thesis we will introduce our equivalent definitions which we think is more appropriate and more beneficially.

Definition of Graph

A graph $G = (V, E)$ is a mathematical structure consisting of two finite sets V and E ,the elements of V are called vertices , and the elements of E are called edge[18] .

A null graph is a graph whose vertex, and edge sets are empty.

A trivial graph is a graph is consisting of one vertex and no edge.

Subgraph a graph H is said to be a subgraph of a graph G iff every vertex in H is also a vertex in G ,every edge in H is also an edge i n G , and every edge in H has the same endpoints as in G.

Types of Graph

A simple graph is a graph that does not have any loops or parallel edges . In a simple graph ,an edge with endpoints V and W is denoted $\{V, W\}$.

A complete graph on n vertices, denoted K_n is a simple graph with n vertices $v_1 \ldots\ldots\ldots v_n$ whose set of edges contains exactly one edg**e** for each pair of distinct vertices[29].

A bipartite graph G is a graph whose vertex-set V can be partitioned into two subsets U, W such that each edge of G has one endpoint in U and one endpoint in W . The pair U, W is called a(vertex) bipartition of G, and U, W are called the bipartition subsets.

A planar graph a graph G is said to be planar if there is drawing of the graph in the plane without edge-crossings

A regular graph is a graph whose vertices all have equal degree.

Line graphs the line graph $L(G)$ of a graph G has a vertex for each edge of G , and two vertices in G are adjacent iff the corresponding edge in G have a vertex in common. Thus, the line graph $L(G)$ is the intersection graph corresponding to the endpoint sets of the edges of G.

Line graphs are special case of intersection graphs.

A directed graph or digraph consists of two finite sets ,a set $V(G)$ of vertices and a set $D(G)$ of directed edges, where each edge is associated with an ordered pair of vertices called its endpoints. If edge e is associated with the pair (V,W) of vertices ,then e is said to be the (directed) edge from V to W [25].

The degree of a vertex is the number of edges that are incident on (or stick out of) the vertex.

Chapter 2
New Graph Theory

Introduction

In this chapter a proposed of a new approach to define and apply the graph theory. This approach is a different form of the classical approach found in the literature. We used this approach to illustrate and represent different circulation problems in networks.

It will be shown that the proposed definition is more appropriate to put the network circulation problems in a more rigorous form. This makes computer-based techniques to solve those problems more adequate and efficient.

2.1-The new definition of graph theory

2.1.1 Definition of a graph:

A graph G is an ordered triple (V, E, ψ) Where:

V: is a finite set called the set of vertices.

E: is a finite set called the set of edges.

ψ :called the incidence function such that

ψ : is a function $\psi : E \rightarrow \zeta$ *where* ζ is a subset of the non-Cartesian product

$V \dot{\times} V$ where the non-Cartesian Product

$A \dot{\times} A = \{\{X, Y\} : X, Y \in A\}$

The numbers of element in V, E are denoted by $|V| = n$, $|E| = m$.

If $\psi(e) = \{v_i, v_j\}$ we call each of v_i, v_j an incident of (e) and they are adjacent to each other.

The third component (ψ) in the above definition was first maintained by Bondy and Murty [7]. But they didn't use it extensively in defining various characteristic of graphs.