شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها
علي هذه الأفلام قد اعدت دون آية تغيرات

يجب أن

تحفظ هذه الأفلام بعيداً عن الغبار

في درجة حرارة من 15 – 20 مئوية ورطوبة نسبية من 20-40 %

To be kept away from dust in dry cool place of
15 – 25c and relative humidity 20-40 %

بعض الوثائق الأصلية تالفة

GEOMETRIC AND ANALYTIC CONSIDERATIONS OF

THE PARALLELISM FOR SUBMANIFOLDS

Thesis

SUBMITTED IN FULFILLMENT FOR THE AWARD
OF THE PH. D. DEGREE IN SCIENCE
(PURE MATHEMATICS)

By

Tarek Fathy Mersal
Department of Mathematics, Faculty of Science
Menoufia University

Submitted to
Faculty of Science, Menoufia University
1998

Supervised by

Prof. Dr. Bernd Wegner
Technical University of Berlin, Institute of Mathematics
Germany

Prof. Dr. Abd El-Shakor Sarhan
Menoufia University, Faculty of Science
Egypt

Prof. Dr. Mohamed Beltagy
Tanta University, Faculty of Science,
Egypt

Dr. Elsaid Lashin
Menoufia University, Faculty of Science, Egypt

ACKNOWLEDGEMENTS

All gratitude is due to God Almighty who guided and aided me to bring-forth to light this thesis. I express heart-felt thanks to my supervisor Prof. Dr. Bernd Wegner, Institute of Mathematics, Technical University of Berlin, Germany, for his suggestion the subject of this work, kind support, continuous help, constructive guidance and valuable discussions throught the development of this thesis. My sincere thanks to all staff members in Institute of Mathematics, TU Berlin, especially Dr. Claus Fieker, Frau Kurtzahn, Frau Hertzberg, Frau Hahnemann, Frau R. Schubert, Frau Jakel and Herrn Andrias.

I am deeply grateful to Prof. Dr. Mohamed Beltagy, Faculty of Science, Tanta University, for his help and hospitality. I am also very grateful to Prof. Dr. A. Sarhan, head of Department of Mathematics, Faculty of Science, Menoufia University, Dr. E. Lashin and all teaching staff members in Depratment of Mathematics, Menoufia University.

Finally, I am very much indebted to all people who gave me the chance to study in Technical University of Berlin, Germany within the channel system and all the people in the Egyptian embassy in Berlin.

Contents

Introduction

The notions of parallel and self-parallel smooth immersions have been introduced by H. R. Farran and S. A. Robertson [9]. A more detailed study of these notions for simply closed curves was done by F. J. Craveiro De Carvalho, and S. A. Robertson in [6] and by B. Wegner in [18] in the general case and in [20] for curves on surfaces.

Our study is an extension to these investigations and an explicit description of the connection of parallelism to the so-called total normal twist of closed and non-closed curves.

In the first chapter we expose and investigate some remarks done by other authors for parallel immersions in Euclidean n-space, and its related notations, especially parallel rank. We give some examples on global parallel rank, covering parallel rank, uniform local parallel rank and local parallel rank satisfying a relation joining these parallel ranks. For plane curves some more detailed results obtained, referring to the connection between parallel rank, curvature and the evolute of the curve. The connection between parallel rank of a smooth immersion of m-dimensional manifold in Euclidean n-space, $m < n$ and the curvature of the normal bundle is explained in detail. The behaviour of parallel rank for cartesian and diagonal product of immersions is studied. At the end of this chapter we discuss the action of conformal transformations on parallel sections of the normal bundle and parallel ranks of an immersion in the Euclidean n-space. This leads to the insight, that all notions and results easily can be transferred to spaces of constant curvature.

In Chapter 2, the total normal twist, special considerations are made for closed curves in Euclidean 3-and 4-space. Parallel transfer along closed curve to a normal vector field along one period of the curve leads to the notation of total normal twist. The classification of self-parallel curves is exhibited and the most important connection of parallel rank to the vanishing of the total normal twist in Euclidean 3- and 4-space is described. This motivates the subsequent study of the total normal twist itself, being connected with the total torsion for Frenet curves. We give some examples to compute the total normal twist for known curves. The behaviour of the total normal twist under the obvious homotopy between a closed curve in the Euclidean 3-space and its plane projection is explained.

Chapter 3 is devoted to studying the variation of the total normal twist. In order to be able to understand the relations between curves of the same total normal twist, variational formulas are developed for this notion. Under special assumptions, we obtain a variational formula for the derivative of the total normal twist depending only on the variational vector field and the data of the original curve. Then Section 3.2 in this chapter is devoted to remove the restriction made for the variational vector field by the special assumptions and to extend the variational formula to general homotopies of closed curves. These results are specialized to the case of regular Frenet curves and extended to curves in Euclidean 4-space. At the end of this chapter we use these results to detect isotopies leaving the total normal twist invariant, and in particular to provide examples for deformations of curves remaining in the class of curves of global parallel rank 2. The given explicit isotopies illustrate the value of the total normal twist. A lot of examples are developed, giving a good intuitive background of this notion and serving as a pool for later examples.

This invites to consider the whole matter in the frame work of infinite-dimensional manifolds. In chapter 4, infinite-dimensional interpretations, we consider the space $C^r(S^1, E^3)$ of all differentiable maps of a circle into the Euclidean 3-space E^3. This space is a Banach manifold. We considered the subspace of all smooth immersions and prove that it is a Banach submanifold of the ambient space. The previous results fit together leading to an interpretation of the total normal twist as a differentiable function.

In Chapter 5, homotopies using conformal transformation, we prove the existence of a homotopy in the Euclidean 3-space from a plane curve to a spherical one preserving the total normal twist. In the work of B. Wegner [19] it has been shown that parallel sections of the normal bundle remain parallel after renormalization, if the ambient space is subject to a conformal transformation and consequently the local and uniform parallel ranks of immersions into Euclidean 3-space is an integer multiple of 2π, then the same is true for any image of the curve under a conformal transformation of the ambient space. Here we use conformal transformations to prove a theorem that for any immersed closed curve c in the Euclidean 3-space with vanishing total normal twist, there exists a Frenet curve homotopic to c in an arbitrary neighbourhood of c such that the total normal twist is invariant along the homotopy between c and the Frenet curve in that neighbourhood.

To obtain more degree of freedom for further constructions, the whole theory of total normal twist is extended to the case of non-closed curves. In Chapter 6 many results obtained previously extend to this generalization. Some additional motivation for these considerations can be obtained from the perspective to carry out analogous investigations for time-like curves in Minkowsky 4-space Furthermore the regularity conditions are reduced to consider a piecewise matching of the non-closed curves to closed ones which are differentiable only. To do this we introduced the notion of relative total normal twist which compares two curves having the same initial point, same end point, same oriented normal plane at the initial point, and same oriented terminal normal plane at the end point, and by using the standard isometries which correspond to each case of locations of the normal plans as given in Section 6.2, we define the notions of norm curves, the total normal twist for non-closed curves. It has been shown that for every case of standard isometry there exists at least one norm curve. in Section 6.4 point out that the degree of smoothness of curves has impact on the behaviour of normal parallel transfer along these curves. We gave an example that assuming C^1 only without having piecewise C^2 will not suffice for a reasonable of normal parallel transfer. Finally we discuss briefly the notion of straight polygons and curved polygons considering them with the previous considerations by matching piecewise C^∞ C^1-curves. Polygons may be taken as a limiting case of these matchings.

These final considerations show that the whole theory can be easily extended to legs regular but nevertheless interesting objects. The extension of the variational formulas will involve some technical efforts.

The important part on the variational formula has been submitted successfully for publication as a joint manuscript to the Proceeding of the Colloquium on Geometry and Topology, held in Braga (Portugal) in September 1997.

Chapter 1

Parallel immersions in Euclidean space

The notions of parallel and self-parallel smooth immersions has been introduced by H. R. Farran and S. A. Robertson [9]. A detailed study of these notions can be found in [6] and in the works of B. Wegner [17], [18], [19] and [20].

The aim of this chapter is to expose and make some remarks about the theory of parallel immersions. Generalities on parallel immersions into Euclidean space are considered. Some general statements about these notions are made. In particular the different kinds of parallel ranks are considered and their relations are made very precise. The connection between parallel rank and the curvature of the normal bundle is explained in detail. The conformal invariance of this rank leads to an immediate transfer of the notations and some results to the ambient space.

1.1 Parallel rank

Let $f : M \longrightarrow E^n$ be a smooth immersion of an m-manifold M into Euclidean n-space, $n - m = k > 0$ is the codimension of M. For any $x \in$

M, the tangent map at x is a linear map $T_xf : T_xM \longrightarrow T_yE^n = E^n$ under the usual identification, $f(x) =: y$. We denote by $L_{f,x} := T_xf(T_xM)$ the image of T_xM as a linear subspace of E^n whose dimension is the rank of f at x, and by $L^N_{f,x}$ its orthogonal complement. The corresponding affine subspaces give the tangent spaces $\tau_f(x) = \{f(x)\} + L_{f,x}$ and the normal spaces $\nu_f(x) = \{f(x)\} + L^N_{f,x}$ to f at x.

Definition 1.1.1 [9] *The two immersions $f, g : M \longrightarrow E^n$ are called parallel, denoted by $f \parallel g$, if for every $x \in M$, $\nu_f(x) = \nu_g(x)$.*

Definition 1.1.2 [9] *A diffeomorphism $J : M \longrightarrow M$ of the domain of $f : M \longrightarrow E^n$ is called a self-parallelism of f, if f and $f \circ J$ are parallel. The self-parallelisms of the immersion f build up a subgroup $G(f)$ of the group of all diffeomorphisms $J : M \longrightarrow M$ under composition. It is called self-parallel group of f.*

We denote by $C = C^\infty(M, E^n)$ the set of all smooth maps $f : M \longrightarrow E^n$. This is a real linear space. Let I_kM denote the subset of C consisting of all immersions of M in E^n.
It is clear that $\parallel$ is an equivalence relation on I_kM. We denote the parallelism class of f by $[f]$. Now we can form the affine hull $A(f)$ of $[f]$ for any f in C. Thus $A(f)$ is the smallest affine subspace of C that contains $[f]$.
As usual let $\bar{\nabla}$ denote the directional derivative in E^n

Proposition 1.1.1 [9] *For all $f \in I_kM$, $[f] = I_kM \cap A(f)$.*

Proof. (i) From the definitions of $[f]$ and $A(f)$ we have that:
for any $f \in I_kM$, $[f] \subset I_kM$ and $[f] \subset A(f)$. Hence $[f] \subset I_kM \cap A(f)$.
(ii) Let the immersions $f_1, ..., f_p \in [f]$, $h = \sum_{i=1}^p t_if_i$ such that $\sum_{i=1}^p t_i = 1$. Then for any $u \in T_xM$ and using that $f_i \parallel f$ for all $i = 1, ..., p$

$$\bar{\nabla}_u h = \sum_{i=1}^p t_i \bar{\nabla}_u f_i = \sum_{i=1}^p t_i (T_xf_i)(u) \in L_{f,x},$$

hence $L_{h,x} \subset L_{f,x}$.
If h is an immersion, then it has rank m, and we obtain $L_{h,x} = L_{f,x}$.