شبكة المعلومات الجامعية

بسم الله الرحمن الرحيم

شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

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لم ترد بالأصـــــل

ON LOCALLY CONVEX SPACES AND OPERATORS ON THEM

THESIS

SUBMITTED FOR THE AWARD OF THE (Ph.D.) DEGREE

IN

(Pure Mathematics)

By

MOHAMED SAAD HASANIN SANAD

UNDER SUPERVISION

Prof. Dr. NASHAT FARIED MOHAMED FATHY

Professor of pure Mathematics

Dept. of Mathematics , Faculty of Science

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Submitted to

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M.S. Sanad

SUMMARY

A topological vector space X is, roughly speaking, a set which carries two structures : a structure of topological space; a structure of vector space.

Furthermore, some kind of compatibility condition must relates these two structures on X. one of the most important classes of topological vector spaces is the class of locally convex spaces. In fact, the theory of locally convex spaces is significantly richer in results than the theory of topological vector spaces, chiefly because there are always plenty of continuous linear functionals on an locally convex space. Moreover, almost all the concrete spaces that occur in functional analysis are locally convex. In a sense, semi--norms and duality play a vital role in the theory of locally convex spaces : In fact, to introduce a topology in a linear space of infinite dimension suitable for applications to classical and modern analysis, it is sometimes necessary to make use of a system of an infinite number of semi-norms. If the system reduces to a single semi-norm, the corresponding linear space is called a normed space. If further more, the space is complete with respect to the topology defined by this semi-norm, it is called a Banach space. Also, duality is what makes this theory powerful because it establishes a tool to translate a problem on the space (where it may appear to be difficult) into one concerning its linear forms (which may happen to be much easier to handle). Duality also admits the replacement of the original topology by simpler ones when dealing with problems involving boundedness convexity, continuity, etc. One of the most important of these topologies is the weak topology on a given locally convex space.

This thesis is devoted to study some cocepts in locally convex spaces, viz., weakly boundedness, duality, some linear topologies (β-topologies, Mackey topologies, τ_{pc} topologies , etc.), equicontinuous sets and compactologies, bilinear forms and topological tensor products.

In a trial to generalize the n-kolmogorov diameters , δ_n (B), of bounded subsets in normed spaces, we arrive to introduce the notions of outer and inner ε-diameters, Δ_n (B,ε) and δ_n (B,ε) , of a bounded subset B. The notion of quasi-compactness of a given subset B is also given. Many results we have obtained in this direction are collected in our first paper which have submitted" for publication under the title "Outer and Inner ε - Diameters of Bounded Sets.

The notions of outer and inner ε- diameters helped us to introduce the notions of diametral numbers of a bounded linear operator. All results we have obtained in this direction are also collected in our second paper which is also submitted for publication under the title .

"Diameteral Numbers of Bunded Linear Operators in Normed Space. "

This thesis consists of five chapters

Chapter (0) :

This chapter contains the basic definitions and notations which we are needed in the thesis.

Chapter 1 :

This chapter is devoted to the study of the theory of locally convex spaces and how we can generate a locally convex space by means of an infinite system of semi-norms. The study of the duality theory for locally convex spaces, and some linear topologies on functional spaces and equicontinuity are our main interests in this chapter.

Chapter 2 :

Our main intersets of study are the properties of the projective topology τ_π defined on the tensors product $X \otimes Y$ of two locally convex spaces X and Y as the strongest locally convex topology on $X \otimes Y$ such that the canonical bilinear map $\otimes : X \times Y \rightarrow X \otimes Y : (x,y) \rightarrow x \otimes y$, is continuous.

A short survey on the algebraic theory of tensor products and the injective norm ε are also contained in this chapter.

Chapter (3) :

In this chapter we introduce the notions of outer and inner ε-diameters, $\Delta_n(B,\varepsilon)$ and $\delta_n(B,\varepsilon)$, of a bounded subset B in a normed space X.

Also we introduce the notion of quasi-compactness of a bounded subset B in a normed space X. Finally we obtain some properties of the outer and inner entropy numbers, $e_n(B)$ and $f_n(B)$, of a bounded set B.

Chapter 4 :

This chapter is devoted to the study of some important classes of compact and nuclear linear operators. We make use of the above notions of outer ε-diameters $\Delta_n(B,\varepsilon)$ and inner ε-diameters $\delta_n(B,\varepsilon)$ to introduce the notions of the diameteral numbers, ε-$D_n(T)$ and ε-$d_n(T)$, of a bounded linear operator T. Also we give a modified definitions of a compact linear operator by means of its diametral numbers ε-$D_n(T)$.

CONTENTS

CHAPTER (0)
SOME NOTATIONS AND BASIC DEFINITIONS

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0.1. Semi- norms and Convex Sets

The semi- norm of a vector in a linear space gives a kind of length for the vector. To introduce a topology in a linear space of infinite dimension suitable for applications to classical and modern analysis, it is sometimes necessary to make use of a system of an infinite number of semi-norms. If the system reduces to a single semi- norm, the corresponding linear space is called a normed linear space. If , furthermore, the space is complete with respect to the topology defined by this semi-norm, it is called a Banach space.

0.1.1. Definition. Let X be a vector space. A real-valued function P defined on X is said to be semi- additive, if for any pair of elements $x , y \in X$,

$$P(x+y) \leq P(x) + P(y),$$

to be positive homogeneous if , for $\lambda \geq 0$,

$$P(\lambda x) = \lambda P(x),$$

and to be homogeneous if, for any λ ,

$$P(\lambda x) = |\lambda| P(x).$$

A semi-additive positive homogneous function is called a gauge function. A homogeneous gauge function is called a semi-norm.