Ain Shams University
Mathematics Department

Faculty of Science

ON SOME CONCEPTS OF FIBREWISE TOPOLOGY

A Thesis

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(Ph.D) in Pure Mathematics (Topology)

By

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Abstract

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The aim of this thesis is to promote the fibrewise viewpoint in some topological structures, namely, topological groups, ideals and grills. Also we introduce the definition of fibrewise group as a beginning of the version of fibrewise algebra and study some of its properties. Moreover Using the concepts of fibrewise Ideal and fibrewise grille a new fibrewise topologies are introduce also we study some properties of these topologies. Finally, we introduce the concept of fibrewise compactness and fibrewise paracompactness in terms of the fibrewise grill.

Summary

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The fibrewise viewpoint is standard in the theory of fibre bundles, however, it has been recognized only recently that the same viewpoint is also of great value in other theories, such as general topology.

In fibrewise topology we work over a topological base space B, say. When B is a one point-space the theory reduces to that of ordinary topology.

To begin with we work in the category of fibrewise set over a given set, called the base set. If the base set is denoted by B then a fibrewise set over B consists of a set X together with a function $p: X \to B$, called the projection.

For each point b of B the fibre over b is the subset $X_b = p^{-1}(b)$ of X; fibres may be empty since we do not require p to be surjective. Also for each subset B' of B we regard $X_{B'} = p^{-1}(B')$ as a fibrewise set over B' with the projection determined by p. The alternative notation X/B' is sometimes convenient.

We regard B as a fibrewise set over itself using the identity as projection. Moreover we regard the Cartesian product $B \times T$, for any set T, as a fibrewise set over B using the first projection.

A fibrewise topological space over B is just a topological space X together with a continuous function $p: X \to B$ called projection and B called the base space, for example the topological product $B \times T$, for any topological space T, can be regarded as a fibrewise topological space over B, using the first projection.

Most of the results obtained so far in this field can be found in James [22] (1984) and James [23] (1991).

For any fibrewise topological space over B the inverse images of the points of B, with respect to the projection, are called fibres. We do not require fibres to be homeomorphic, as they are in fibre bundle theory.

The aim of this thesis is to promote the fibrewise viewpoint in some topological structures, namely, topological groups, Ideals and Grills. In fact there appear to be fibrewise versions of these concepts.

Moreover the version of fibrewise abstract algebra is first introduced in this thesis by giving the definition and the basic properties of the fibrewise group, which is needed for obtaining and studying the fibrewise version of topological group .

In 1933, kuratowski[37] introduced the concepts of an ideal on a nonempty set ,also several authors are interested in this line of study and therefore some sorts of ideals arise as one goes futher in mathematics such as the ideal of finite subsets of X, the ideal of nowhere dense sets and the ideal of meager sets.

Vaidyanathaswamy in 1945 [59] introduced the concept of a local function and a kuratowski closure operator which were considered as the main-entrance of the concepts of ideal into the topological spaces,

In 1947, Choquet[9] introduced the concept of grills, which was subsequently found to be an extremely useful device, like filters and nets, for the investigations of many topological notions like compactifications, proximity spaces, different typs of extension problems (see [8, 57]). In [52], Roy and Mukherjee gave the formulation of a new topology on a given topological spaces, constructed from the existing topology of the space and a given grill. In [51] Roy and Mukherjee introduced a kind of compactness property, termed as $\mathcal{G}-$compactness, in a topological space (X,τ), defined in terms of a grill $\mathcal{G}$ on X, and in [13] Hamlett and Jankovic defined the $I-$compactness in a topological space X corresponding to an ideal I on the underlying set X. In fact, Kandil, El-Sheikh and other proved in [33] that , the topology

induced by an ideal and the topology induced by a grill are equivalent. Also the I −compactness and the $\mathcal{G}-$ compactness are equivalent.

In 2009 Roy and Nath Mukherjee [50] introduced a kind of paracompactness in a topological space via grills. Such an endeavour gives rise to a generalized version of paracompactness called $\mathcal{G}$ −paracompactness. A similar attempt towards such generalization under the terminology I-paracompactness" was undertaken by Hamlett et al [15].

This thesis consists of six chapters.

Chapter one is an introductory considered as background for the material included in this thesis. It contains basic concepts, definitions, and propositions of fibrewise topology, basic definitions and some theorems of topological groups, also basic definitions and properties for the mathematical structures of ideals and grills. Also, some topological concepts in terms of ideals and grills were recalled

Chapter two consists of three sections. In section one; we introduce the concepts of fibrewise group, fibrewise subgroup and fibrewise homomorphism. Also we proved several new results concerning it. In section two; we define the fibrewise direct product of fibrewise groups and study some of its properties. In section three; we introduce the concept of

fibrewise topological group, some examples are given, many important and intersected properties and new results are proved.

Chapter three consists of two sections. In section one; we introduce and study the fibrewise separation axioms in fibrewise topological groups and investigate the relations between the fibre over the identity and the space. In section two; we introduce the fibrewise compactness in fibrewise topological groups and prove some of its properties.

Chapter four consists of three sections. In section one; we define of fibrewise ideals on a nonempty fibrewise set and study some of its properties. In section two; we define the fibrewise local function for a fibrewise space X over B is defined using fibrewise ideal on X, this function generates a fibrewise topology from the given fibrewise topology on X. In section three; we restrict the definition of the above local function on each fibre X_b over b where $b \in B$ using a fibrewise ideal, and we study its properties.

Chapter five consists of three sections. In section one; we give the definition of fibrewise grill and its properties. In section two; a fibrewise topology is generated by fibrewise grill and we further show the fibrewise topology induced by fibrewise ideal and the fibrewise topology induced by fibrewise

grill are equivalent. In section three; we have continued to study the properties of fibrewise topology induced by fibrewise grill. We defined and introduced the concept of fibrewise compactness based on a fibrewise grill in a fibrewise topological space. Also we study the properties of such fibrewise spaces and the relationships between them using by fibrewise map.

The aim of chapter six is to define the fibrewise $\mathcal{G}-$paracompactness and fibrewise almost paracompactness and also we study the properties of such fibrewise spaces and the relationships between them using by fibrewise map.

Some of the main results in this thesis were published in [48 & 58], other results were presented with scientific journal

CHAPTER (I)

Preliminaries

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Preliminaries

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1.1. Fibrewise topology

Definition 1.1.1 [23]: Let B be any set. Then a fibrewise set over B consists of a set X together with a function $p: X \to B$, called the projection, and B is called the base set.

For each $b \in B$, the fibre over b is the subset $X_b = p^{-1}(b)$ of X. Also for each subset W of B, we regard $X_W = p^{-1}(W)$ is a fibrewise set over W with the projection determined by p.

Proposition 1.1.2 [23]: Let X be a fibrewise set over B, with projection p. Then Y is a fibrewise set over B with projection $p\alpha$ for each set Y and function $\alpha : Y \to X$.