University College for Women

APPLICATIONS IN LIFE TESTING: PROGRESSIVE CENSORING FOR SOME DISTRIBUTIONS

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Abstract

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In this thesis, the progressively type-II right censored samples for some useful distributions are considered.

In the first part we introduce definition, notations of order statistics and progressive censoring.

In the second part, of this thesis, we establish some recurrence relations satisfying by the single and product moments of progressive Type-II right censored order statistics from the left and doubly truncated Lomax distributions.

In the third part, some new recurrence relations satisfied by the single and product moments of progressively censored sample for the double truncated power function distribution are presented.

The fourth part concerned with recurrence relations for the single and product moments of progressive Type-II right censored order statistics from the inverse Weibull, the left and the doubly truncated inverse Weibull.

Key words: order statistics, censoring, progressive Type-II censoring, recurrence relations, single moments, product moments, and power function, Lomax and inverse Wiebull distributions.

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Chapter I

Introduction

Order statistics and their applications have been studied rather extensively since the early part of the last century. The statistical analysis of what is variously referred to as failure time, lifetime or survival data has widely developed, especially in the biomedical and engineering sciences. Applications of survival analysis range from research involving human diseases to investigations in the endurance of manufactured items. Survival data often come with a special feature. It is known as censoring and occurs when exact survival times are known only for a portion of the individuals or items under study. The complete survival times are not observed by the experimenter either intentionally or unintentionally. The planning of experiments with the aim of reduce both of the number of failures and the total duration time of the experiment leads naturally to the will-known type-I and type-II right censoring schemes. The generalization of type-I and type-II censoring is called progressive censoring scheme. Progressively Type-II right censored order statistics is a natural generalization of the usual order statistics [see Lawless 1982, Balakrishnan and Cohen 1991].

Suppose that(x_1, x_2, …, x_n) are n jointly distributed random variables. The corresponding order statistics are the x_i's arranged in non-decreasing order. The smallest of the x_i's is denoted by $x_{1:n}$, the second smallest is denoted by $x_{2:n}$, …, and, finally, the largest is denoted by $x_{n:n.}$ Thus $x_{1:n} \le x_{2:n} \le \ldots \le x_{n:n}$ is the ordered sample of size n. [for more details see David and Nagaraja 2003, Arnold et al.1992]

1.1 Distribution of Order Statistics

Let us assume that x_1, x_2 , …x_n is a random sample from an absolutely continuous population with probability density function f(x) and cumulative distribution function F(x) ; let $x_{1:n} \le x_{2:n} \le \ldots \le x_{n:n}$ be the order statistics obtained by arranging the preceding random sample in increasing order of magnitude. The density function of the i-th order statistic $x_{1:n}$ from the continuous distribution f(x) has its pdf as

$$f_{i:n}(x) = \frac{n!}{(i-1)!(n-i)!}[F(x)]^{i-1}[1-F(x)]^{n-i} f(x), \qquad -\infty < x < \infty. \quad (1.1)$$

Similarly, the joint probability density function of the i-th and j-th order statistics $x_{i:n}$ and $x_{j:n}$ ($1 \le i \le j \le n$) is given by

$$f_{i,j:n}(x_i, x_j) = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}[F(x_i)]^{i-1}[F(x_j)-F(x_i)]^{j-i-1} \\ \times[1-F(x_j)]^{n-j} f(x_i) f(x_j), \qquad -\infty < x_i < x_j < \infty. \quad (1.2)$$

1.2 Moments of Order Statistics

The k-th single moment of the i-th order statistic denoted by $\mu_{i:n}^{(k)}$ is given by

$$\mu_{i:n}^{(k)} = \frac{n!}{(i-1)!(n-i)!}\int_{-\infty}^{\infty} x^k [F(x)]^{i-1}[1-F(x)]^{n-i} f(x)dx, \qquad i=1,\dots,n,\ k\geq 1. \quad (1.3)$$

The product (k_1, k_2)-th moments of $X_{i:n}$ and $X_{j:n}$ denoted by $\mu_{i,j:n}^{(k_1,k_2)}$, k_1, k_2=0, 1, 2, …, is given by

$$\mu_{i,j:n}^{(k_1,k_2)} = \frac{n!}{(i-1)!(j-i-1)!(n-j)!}\int_{-\infty}^{\infty}\int_{x_i}^{\infty} x_i^{k_1} x_j^{k_2} [F(x_i)]^{i-1}$$
$$\times [F(x_j)-F(x_i)]^{j-i-1}[1-F(x_j)]^{n-j} f(x_i) f(x_j) dx_j dx_i.$$
$$1 \leq i < j \leq n \qquad (1.4)$$

see [Arnold et al. 1992].

1.3 Types of Censoring

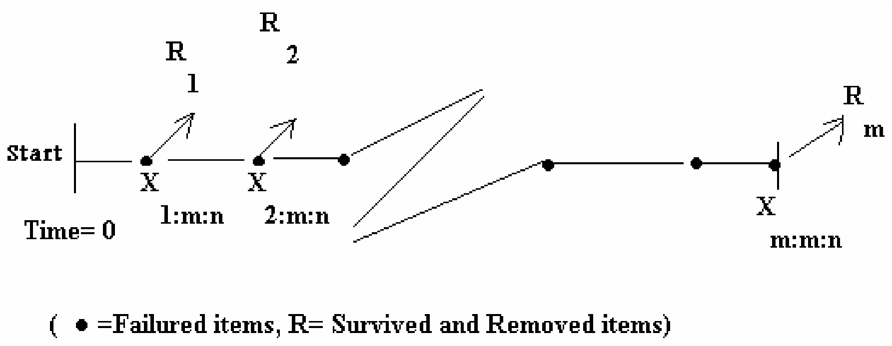
Consider a life-testing experiment where n items are kept under observation until failure. Suppose the life lengths of these n items are i.i,d random variables. For some reason or other, suppose that we have to terminate the experiment before all items have failed. We would then have a censored sample.

Now we have two types of censored, If the experiment is terminated at a predetermined time t, so that only the failure times of the items that failed prior to this time are recorded. The data so obtained constitute a Type-I censored sample. Clearly, the number of observed order statistics is a random variable. If the experiment is terminated at

the r-th failure that is we obtain Type-II censored sample. The duration of the experiment is random variable.
The generalization of type-I and type-II censoring is called progressive censoring scheme.

In many life testing studies, it is common that lifetimes of some test units may not be able to record exactly. In Type-II censoring, the test cause of testing order to save time or cost. Furthermore, some test units may have to be removed at different stages in the study for various reasons. This would lead to progressive censoring. In some cases when there are live units on test intentional removal of units or termination of an experiment may be due to ethical considerations. Conventional type-I and type-II one-stage right censoring has been studied by many authors including Lawless (1982), Nelson (1982), Cohen (1991) and London (1988), Who consider life time studies in industrial as well as actuarial contexts, In both parametric and non-parametric cases. Consider a sample of n units put on a test at time 0, in type-I right censoring, a time t, independent of the failure times and it is fixed such that beyond this time no failures are observed, that is, experimentation terminates. Thus the number of complete life times (and therefore the number of partial life time) observed is a random variable. Type-II censoring differs in that the number of observed failures is fixed; say m, so that at the time of the m-th failure, experimentation terminates leaving n-m partially observed failure times. Here, time of termination of the experiment is random. Both of type-I and type-II one-stage right censoring described above have been generalized to the case of double (left and right) censoring، where in the case of type-I censoring, observation begins at a fixed time t_L rather than at time 0, and in type-II censoring, observation of failures begins with the (r+1)-the failure where r is fixed, rather than at time 0. These situations arise when, for example the first few failures occur too quickly to be observed [see Nelson (1982)].

Progressive Type-II right censoring has been introduced in context of life testing by Cohen (1963) in the following manner. After starting the life testing experiment with fixed sample size n units, a number of complete observations m and a censoring scheme $(R_1, R_2, ..., R_m)$ and $(n = m + \sum_{i=1}^{m} R_i)$. The n units are placed simultaneously on life test. Arise as follows, immediately following the first failure, R_1 surviving units randomly chosen are removed from the experiment. Then immediately following the second failure, R_2 surviving units randomly chosen are removed from the experiment. This process is continuous until, at the time of the m-th failure, all the remaining $R_m = n - R_1 - R_2 - ... - R_{m-1} - m$ units are removed from the experiment. Here the R_i's are fixed prior to study. If $R_1 = R_2 = ... = R_m = 0$ then n=m which corresponds to complete sample situation. If $R_1 = R_2 = ... = R_{m-1} = 0$, we have $R_m = n - m$ which corresponds to the conventional Type-II right censoring [see for more details Mann(1969,1971), Balakrishnan and Aggarwala (2000)]. This is a generalization of the Type-I right censoring, or Type-I multi-stage right censoring. A generalization of the conventional Type-II one stage right censoring is considered in this thesis. It is referred to as progressive Type-II right censoring. This censoring scheme may be depicted pictorially as follows:



Mann (1969, 1971), Thomas and Wilson (1972) and Cacciari and Montanari (1987) have discussed some linear inferences for the case of progressive type-II right censoring when the life time distributions are Weibull and exponential distributions. Viveros and Balakrishnan (1994) have proposed conditional method of inference used to derive exact confidence intervals for several life characteristics such as location, scale, quantiles, and reliability when the data are type-II progressively censored. Aggarwala and Balakrishnan (1998) have established some properties of the progressive type-II censored order statistics from arbitrary continuous distributions. They used these properties to develop an algorithm to simulate a general progressive type-II censored order statistics from any continuous distributions, by generalizing the algorithm given recently by Balakrishnan and sandhu (1995), they have established an independence result for general progressive type-II censored samples from the standard uniform distribution. This result is used in order to obtain moments of general progressive type-II censored order statistics from the standard uniform distribution. Finally, they have derived the best linear unbiased estimators (BLUEs) of the parameters of one- and two-parameters uniform distributions as well as the discussed problem of the maximum likelihood estimation is discussed.

Saleh (2005) has established several recurrence relations for single and product moments for Lomax, Weibull, linear exponential and half logistic distribution. Then he derived exact form of best linear unbiased estimators (BLUES) for the location and scale parameters of Lomax distribution. See also sultan et al. (2005, 2006 and 2007). Thomas and Wilson (1972) have developed computational method that allows one to compute the single and product moments of order statistics from progressively censored samples by making use of the corresponding moments of the order statistics from the complete sample. The absence of an explicit representation for the marginal and joint of density function of order statistics under progressive censoring makes their method extremely tedious. By driving the required marginal and joint density functions in explicit forms, Balakrishnan et al. (2002) have obtained an alternative, highly efficient, method for computing the desired moments. Fernández (2004) has discussed the problem of estimation of parameters of exponential, on the basis of the general progressive type-II censored sample, using both classical and Bayesian view points. Bordes (2004) has consider nonparametric estimation of the cumulative hazard functions and reliability function of progressively Type-II right censored data, he has showed that the nonparametric maximum likelihood estimators can be derived under such censoring schemes. These estimators are obtained in the reliability context but they can also be extended to arbitrary continuous distribution function, he compared the non-parametric estimators of the reliability with two parametric estimators based on a real data set.

Aggarwala and Childs (1999) have developed procedures for obtaining confidence intervals of the location and scale parameters of Pareto distribution as well as the upper and lower γ probability tolerance intervals of the proportion β when the observed samples are progressively censored. Balakrishnan and Sandhu (1996) have derived the best linear unbiased estimators for the parameters of one- and tow-parameters exponential distributions based on general progressive Type-II censored samples and the maximum likelihood estimators. They also gave an example to illustrate their technique of estimation. Shuo-Jye Wu (2002) has obtained the maximum likelihood estimates of the shape and scale parameters based on concerning progressively Type-II censored sample from the Weibull distribution. Also, he has constructed an exact confidence intervals and an exact confidence region of the shape and scale parameters. Balakrishnan and Rao (2003) have shown that the best linear unbiased estimators of the location and scale parameters of a location-scale parameter distribution based on a general progressive Type-II censored sample are in fact trace-efficient linear unbiased estimators as well as determinant-efficient linear unbiased estimators. More generally, they have shown that the best linear unbiased estimators possess complete covariance matrix dominance in the class of all linear unbiased estimators of the location and scale parameters. Ali Mousa and Jaheen

(2002) have obtained the maximum likelihood and Bayes estimates of the two parameters and the reliability function of the Burr Type XII distribution based on progressive Type-II censored samples.

Balakrishnan, et al. (2001) have derived bounds for expected values and variances of progressive Type-II censored order statistics by applying different methods. Numerical examples presented to compare bounds and exact values for means w.r.t. underlying rectangular and normal distributions. Malik, et al. (1988) have reviewed several recurrence relations and identities available for the single and product moment of order statistics from an arbitrary continuous distribution, and they have pointed out the inter relationship between many of these recurrence relations. They have reviewed several recurrence relations and identities established for the single and product moments of order statistics from specific continuous distributions. Balakrishnan and Sultan (1998) have updated the reviews of Balakrishnan, et al. (1988) discuss several recurrence relations and identities for moments and product moments of order statistics. N. Balakrishnan, Erhared Cramer (2006) has introduced the progressively censored order statistics from heterogeneous distributions and they have investigated their properties. N. Balakrishnan and P.S. Chan (1992) have derived the best linear unbiased estimator (BLUE) based on doubly Type-II censored samples for the scaled half logistic distribution. Indrani Basak and N. Balakrishnan (2003) have derived the influence function and the breakdown point of robust M-estimators for progressively censored failure time data, they have developed the most robust and the optimal robust estimators, and they have characterized the optimal