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Department of Mathematics

INFERENCE FOR STRESS-STRENGTH MODELS BASED ON BIVARIATE EXPONENTIAL DISTRIBUTIONS

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CONTENTS

Acknowledgements **I**
Abstract **II**
Summary **III**

Chapter 1
Introduction and Basic Concepts

1.1. Introduction. 1
1.2. Literature review 2
1.3. Review on bivariate exponential distribution 8
1.4. System reliability 14
1.5. Stress-strength models. 17
1.6. Statistical estimation 20

Chapter 2
Freund's Exponential Distributions

2.1. Introduction 25
2.2. Derivation of the Freund's bivariate exponential distribution. 27
2.3. Statistical properties of the bivariate model. 28
2.4. Freund -Weinman multivariate exponential distribution. 37
2.5. Freund -Weinman-Block multivariate exponential distribution 38
2.6. Distribution function of ordered variables 44

Chapter 3
Marshall-Olkin Exponential Distributions

3.1. Introduction 51
3.2. Derivation of the BVE distribution. 52
3.3. Properties of the BVE distribution. 57
3.4. The multivariate exponential distribution. 61
3.5. Estimation of the parameters of Marshall-Olkin exponential distributions 66

Chapter 4
Reliability of Stress-Strength Systems Based on the Freund's Bivariate Exponential Distribution and its Extension

4.1. Introduction 79
4.2. Reliability of the system subjected to a common stress. 81
4.3. Reliability of the system subjected to two stresses 84
4.4. Reliability of the system subjected to multivariate stresses 89
4.5. Reliability of the system subjected to n independent stresses. 92
4.6. Estimation of the reliability. 103
4.7. Numerical illustration. 106

Chapter 5
Reliability of Stress-Strength Systems Based on the Marshall-Olkin Bivariate Exponential Distribution and its Extension

5.1. Introduction. 113
5.2. Reliability of the system subjected to a common stress 115
5.3. Reliability of the system subjected to two stresses 118
5.4. Reliability of the system subjected to multivariate stresses. 125
5.5. Reliability of the system subjected to n independent stresses 127
5.6. Estimation of the reliability. 135
5.7. Numerical illustration. 136

References. 149

Abstract

Sohair Khames Khames Gomaa.
Inference for stress-strength models based on bivariate exponential distributions Master of Science Dissertation of Pure Mathematics, University College for Women, Ain Shames University.

The stress-strength models are of substantial interest and are useful in various branches of science such as reliability theory, psychology and medicine. The concept of reliability of the stress-strength models is considering the strength of a system and the stress to be encountered as random variables. The reliability of a system is the probability that it is strong enough to overcome the stress imposed on it. In this thesis we study stress-strength models based on bivariate exponential distributions and their extensions. Some well-known bivariate exponential distributions, that are applicable in the reliability theory, are presented. Two distributions, namely Freund and Marshall-Olkin bivariate exponential distributions are studied extensively with their extensions. The maximum likelihood estimators of the parameters of the Freund-Weinman-Block exponential distribution are derived using the ordered variables of the distribution. The reliability of some parallel and series multi-component stress-strength models is studied. The strengths of the components are either the Freund-Weinman-Block exponential distribution or the $(k+1)$-parameter Marshall-Olkin exponential distributions. Different stress-strategies are discussed. A common stress or a number of stresses could be imposed on the system. The stresses could be either independent or dependent. For the dependent case, the stresses are assumed to have either the Freund-Weinman-Block exponential distribution or the $(k+1)$-parameter Marshall-Olkin exponential distributions. In this case it is assumed that any stress could be imposed on any component. For the independent case, the stresses are assumed to be non-identically and exponentially distributed. Also for the case of independent stresses, it is assumed that a certain stress is imposed on a certain component. The case of additive independent stresses is also studied. For all models considered, the reliability function is derived. Using different methods of estimation different estimators of the reliability function of each model are obtained. Finally, numerical illustrations are performed by means of a simulation study to highlight the theoretical results obtained.

Summary

The stress-strength models are of substantial interest and are useful in various branches of science such as reliability theory, psychology and medicine. The concept of reliability of the stress-strength models is considering the strength of a system and the stress to be encountered as random variables. The reliability of a system is the probability that it is strong enough to overcome the stress imposed on it. In this thesis we study stress-strength models based on bivariate exponential distributions and their extensions.

The thesis consists of five chapters:

Chapter 1
Introduction and Basic Concepts

Chapter 1 presents a brief literature review concerning the point of the research and a description of the thesis. Some well-known bivariate exponential distributions that are applicable in the reliability theory are presented. The definitions and the basic concepts that are used throughout the thesis are also introduced.

Chapter 2
Freund's Exponential Distributions

In Chapter 2, we study the Freund's bivariate exponential distribution and its extensions. The derivation and the statistical properties of the Freund's bivariate exponential distribution are given. Extensions of the Freund's bivariate exponential distribution are presented, namely Freund-Weinman and Freund-Weinman-Block multivariate exponential distributions.
The joint density and the marginal distributions of the ordered variables of the Freund-Weinman-Block multivariate exponential distribution are obtained. Using the ordered variables, the maximum likelihood estimators of the parameters of the Freund-Weinman-Block multivariate exponential distribution are derived. The results of this chapter are accepted for publication in "Ninth International Conference on Ordered Statistical data and Their Applications" Faculty of Science, Zagazig University, Egypt (2010).

Chapter 3
Marshall-Olkin Exponential Distributions

In Chapter 3, we study the Marshall-Olkin bivariate exponential distribution and its extensions. The derivation of the Marshall-Olkin bivariate exponential distribution and its statistical properties are presented. The generalizations of the Marshall-Olkin bivariate exponential distribution with the $(2^k - 1)$, and the $(k+1)$ parameters, respectively, are presented.
Different estimation methods of the parameters of the bivariate and the $(k+1)$-parameter Marshall-Olkin exponential distributions are discussed.

Chapter 4
Reliability of Stress-Strength Systems Based on the Freund's Bivariate Exponential Distribution and its Extension

In Chapter 4, we study the reliability of some parallel and series multi-component stress-strength models. The system consists of k, $k \geq 2$ components. The strengths of the components are assumed to have the Freund-Weinman-Block multivariate exponential distribution. This distribution is an extension of the Freund's bivariate exponential distribution. The system is subjected to different stress-strategies as follows:

1. The components of the system are subjected to an exponential common stress.
2. The components of the system are subjected to 2 different dependent stresses. The stresses are assumed to have the Freund's bivariate exponential distribution. In this case two assumptions are studied:
   (i) Each component is subjected to any one of the 2 stresses.
   (ii) The system is subjected to an additive stress, which is the sum of the 2 stresses.
3. The components are subjected to n different dependent stresses. Each component is subjected to any one of the n stresses. The stresses are assumed to have Freund-Weinman-Block multivariate exponential distribution.
4. The components are subjected to n non-identical independent exponential stresses. Here the following assumptions are considered:
   (i) Each component is subjected to any one of the n stresses.
   (ii) Each component is subjected to a specific stress, $n = k$.
   (iii) The system is subject to the total sum of the n stresses.

For all cases the stresses and the strengths are assumed to be independent.
The reliability functions of all the above models are derived. Estimation of the reliability of each model is obtained using parametric and non-parametric methods. For the parametric method, using the maximum likelihood estimates of the parameters, obtained in Chapter 2, the maximum likelihood estimate of the reliability is obtained. Non-parametric estimation of the reliability is obtained by estimating the probability that the strength of the system is greater than the stress imposed on it.
Finally, numerical illustrations are performed by means of a simulation study to highlight the theoretical results obtained. A comparison is presented between reliability of the system under the different cases of stresses. Also, a comparison between the parametric and the non parametric reliability estimates is introduced. The results of this chapter are accepted for publication in "Ninth International Conference on Ordered Statistical data and Their Applications" Faculty of Science, Zagazig University, Egypt (2010).

Chapter 5
Reliability of Stress-Strength Systems Based on the Marshall-Olkin Bivariate Exponential Distribution and its Extension

In Chapter 5, we study the same stress-strength models described in Chapter 4, considering the Marshall-Olkin bivariate exponential distribution and its extension, the $(k+1)$ -parameter Marshall-Olkin exponential distribution.

The reliability function of each model is derived. Parametric estimation of the reliability models is obtained. Using the different methods of estimation of the parameters given in Chapter 3, different reliability estimates are obtained. Also, as in Chapter 4, non-parametric reliability estimates are obtained.
A numerical illustration is performed at the end of the chapter. Again a comparison is presented between system reliabilities under the different stress strategies. Also a comparison between different reliability estimates is introduced.

Chapter 1

Introduction and Basic Concepts

1.1 Introduction

The stress-strength models, are of substantial interest and useful in various branches of science such as reliability theory, psychology and medicine. The concept of reliability of the stress-strength models is considering the strength of a system and the stress to be encountered as random variables, say X and Y respectively. The system fails if and only if at any time the applied stress exceeds the strength ($Y > X$). Thus the reliability of the system is defined as the probability of not failing,

$$R = P(X > Y).$$

The problem is to estimate the reliability of the system R where both X and Y are random variables. There is a substantial number of papers devoted to probabilistic problems associated with evaluation of R and construction of efficient estimators of this parameter based on sample values with various assumptions on the distributions