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شبكة المعلومات الجامعية

التوثيق الالكتروني والميكروفيلم



شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

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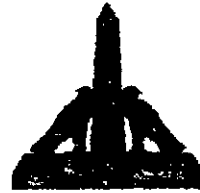
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Role of Legendre Polynomial in Some Physical Problems

**A thesis
Submitted in the partial fulfillment for
M.Sc degree in physics**

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2002

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﴿ الحمد لله الذي هدانا لهذا وما كنا

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English Summary

Summary

In this thesis a brief review of some special functions interesting in physics, such as Γ , Hermite, and Legendre functions.

In the introduction a historical review of associated Legendre polynomial, its responsibility of appearance of two quantum numbers L, m and their complex expressions.

In chapter I description of Γ function as Euler definition, Weierstrass and Factorial definitions.

Also description of Hermite polynomial, its application in harmonic oscillator problem.

In chapter II details of properties of Legendre polynomials; orthogonal, normalization; and how any function $F(z)$ could be represented as a power series in terms of Legendre polynomial.

In chapter III a brief presentation of some applications of Legendre polynomial in physics, e.g. in scattering theory in high energy physics and gravitation.

We have chosen two examples to describe in detail.

One of them depends on the generating function of Legendre polynomial (in electromagnetic theory).

The other the Legendre polynomial does not appear explicitly, this is the Zeeman effect, where it could be explained by the idea of complex magnetic quantum number $m = m_L \pm im_S$.

Introduction