



# **SYSTEM FREQUENCY TUNING FOR HEAVING BUOY WAVE ENERGY CONVERTERS**

By

**Ahmed Hamdy Abdelmaguid Sakr**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
**Mechanical Design and Production Engineering**

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2017

# **SYSTEM FREQUENCY TUNING FOR HEAVING BUOY WAVE ENERGY CONVERTERS**

By

**Ahmed Hamdy Abdelmaguid Sakr**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
**Mechanical Design and Production Engineering**

Under the Supervision of

**Prof. Dr. Sayed M. Metwalli**

.....

Professor Emeritus

Mechanical Design and Production Engineering Department  
Faculty of Engineering, Cairo University

**Assoc. Prof. Yasser H. Anis**

.....

Associate professor

Mechanical Design and Production Engineering Department  
Faculty of Engineering, Cairo University

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2017

# **SYSTEM FREQUENCY TUNING FOR HEAVING BUOY WAVE ENERGY CONVERTERS**

By

**Ahmed Hamdy Abdelmaguid Sakr**

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
**Mechanical Design and Production Engineering**

Approved by the Examining Committee:

---

<b>Prof. Dr. Sayed M. Metwalli</b>	<b>Thesis Main Advisor</b>
------------------------------------	----------------------------

---

<b>Assoc. Prof. Yasser H. Anis</b>	<b>Advisor</b>
------------------------------------	----------------

---

<b>Prof. Dr. Saad A. Kassem</b>	<b>Internal Examiner</b>
---------------------------------	--------------------------

---

<b>Prof. Dr. Mustafa H. Arafa</b> <b>The American University in Cairo</b>	<b>External Examiner</b>
--	--------------------------

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2017

**Engineer's Name:** Ahmed Hamdy Abdelmaguid Sakr  
**Date of Birth:** 26/08/1990  
**Nationality:** Egyptian  
**E-mail:** a.hamdy@cu.edu.eg  
**Phone:** +201063624828  
**Address:** Faculty of Engineering, Cairo University  
**Registration Date:** 1/10/2012  
**Awarding Date:** / /2018  
**Degree:** Master of Science  
**Department:** Mechanical Design and Production Engineering



**Supervisors:**

Prof. Dr. Sayed M. Metwalli  
Assoc. Prof. Yasser H. Anis

**Examiners:**

Prof. Dr. Mustafa H. Arafa	(External Examiner)
The American University in Cairo	
Prof. Dr. Saad A. Kassem	(Internal Examiner)
Prof. Dr. Sayed M. Metwalli	(Thesis Main Advisor)
Assoc. Prof. Yasser H. Anis	(Advisor)

**Title of Thesis:**

System Frequency Tuning for Heaving Buoy Wave Energy Converters

**Key Words:**

Wave energy; Heaving; Buoy; Energy frequency; Reactive loading control

**Summary:**

In this thesis, we present a quick long-term reactive loading control for heaving buoy wave energy converter. The control is achieved by introducing an external continuous variable stiffness that is connected to the buoy to tune its natural frequency. The external stiffness is connected to the buoy through a continuous V-belt drive to change its effect on the buoy. This control maximizes the oscillation amplitudes; hence, maximizing the power absorption efficiency by achieving near-resonance operation. Analytical results show a promising power absorption increase as compared to other control techniques. Experimental results, for an equivalent vibrating system, prove the concept of the control operation.

# Acknowledgements

I would like to express my deepest appreciation and gratitude to my supervisors, Prof. Dr. Sayed M. Metwali and Assoc. Prof. Yasser H. Anis, for their supervision of this work. I appreciate their continuous and invaluable academic support and belief in me as a researcher.

I would like to acknowledge my dear colleagues, Eng. Khalid M. Abdelaziz and Eng. Kirolos M. Henry, for their support in design and manufacturing of the experimental setup.

# Dedication

This thesis work is dedicated to my wonderful parents who have been a constant source of support and encouragement during the challenges of graduate school and life.

# Table of Contents

<b>ACKNOWLEDGEMENTS</b>	<b>i</b>
<b>DEDICATION</b>	<b>ii</b>
<b>TABLE OF CONTENTS</b>	<b>iii</b>
<b>LIST OF TABLES</b>	<b>vi</b>
<b>LIST OF FIGURES</b>	<b>vii</b>
<b>NOMENCLATURE</b>	<b>ix</b>
<b>ABSTRACT</b>	<b>xiv</b>
<b>CHAPTER 1: INTRODUCTION AND LITERATURE REVIEW</b>	<b>1</b>
1.1 BACKGROUND . . . . .	1
1.1.1 Wave development . . . . .	2
1.1.2 Converters location . . . . .	2
1.1.3 Converter types . . . . .	3
1.1.4 Power take-off systems . . . . .	5
1.1.5 Control . . . . .	9
1.2 LITERATURE REVIEW . . . . .	10
1.2.1 Reactive control theory . . . . .	10
1.2.2 Latching and declutching control theory . . . . .	12
1.2.3 Reactive loading control for heaving buoys . . . . .	14
1.3 SCOPE OF WORK . . . . .	15
<b>CHAPTER 2: MATHEMATICAL MODELLING</b>	<b>16</b>
2.1 SEA STATE MODELLING . . . . .	16
2.1.1 Pierson-Moskowitz spectrum . . . . .	16
2.1.2 Pierson-Moskowitz spectrum calculations . . . . .	18

2.1.3	Irregular waves simulation . . . . .	18
2.2	GOVERNING EQUATION . . . . .	21
2.3	RADIATION FORCE MODELLING . . . . .	22
2.3.1	Computation of hydrodynamic coefficients . . . . .	23
2.4	EXCITATION FORCES MODELLING . . . . .	24
2.5	STATE SPACE MODELING AND SOLUTION . . . . .	25
2.5.1	The approximation of a convolution integral by a state-space model	25
2.6	POWER ABSORPTION AND EFFICIENCY . . . . .	28
<b>CHAPTER 3: SYSTEM TUNING</b>		<b>29</b>
3.1	DESCRIPTION OF THE CONTROLLER . . . . .	29
3.2	MATHEMATICAL MODEL OF THE CONTROLLER MECHANISM . .	31
3.2.1	The mechanical impedance of oscillating systems . . . . .	31
3.2.2	V-belt dimensions . . . . .	32
3.2.3	Sheaves running diameters . . . . .	32
3.2.4	Sheaves axial position . . . . .	33
3.2.5	Inertia effect of CVS mechanism . . . . .	34
3.3	CONTROLLER OPERATING BANDWIDTH . . . . .	34
3.4	OPTIMUM PTO DAMPING . . . . .	36
3.4.1	Radiation absorption . . . . .	37
<b>CHAPTER 4: ANALYTICAL AND EXPERIMENTAL RESULTS</b>		<b>39</b>
4.1	SYSTEM ANALYTICAL RESULTS . . . . .	39
4.1.1	Excitation forces . . . . .	39
4.1.2	Regular waves . . . . .	39
4.1.3	Irregular waves . . . . .	43
4.2	EXPERIMENTAL SETUP . . . . .	46
4.2.1	Exciter mechanism . . . . .	48
4.2.2	CVS mechanism . . . . .	48
4.2.3	CVS system speed ratio . . . . .	48



4.3 EXPERIMENTAL RESULTS . . . . .	51
<b>CHAPTER 5: CONCLUSION</b>	<b>55</b>
<b>REFERENCES</b>	<b>56</b>
<b>APPENDIX A: ANALYTICAL SOLUTION ALGORITHMS</b>	<b>59</b>
<b>APPENDIX B: EXPERIMENTAL SETUP DRAWINGS</b>	<b>83</b>
<b>APPENDIX C: RANDOM TO DETERMINISTIC TRANSFORM</b>	<b>108</b>
C.1 RANDOM TO DETERMINISTIC TRANSFORM . . . . .	108
C.2 SOLUTION ALGORITHMS USING RANDOM TO DETERMINISTIC TRANSFORM . . . . .	114

# List of Tables

1.1	Comparison between different WEC types . . . . .	8
3.1	Selected system parameters and V-belt dimensions . . . . .	33
4.1	Experimental setup parameters . . . . .	47

# List of Figures

1.1	Surface waves generated by wind [1] . . . . .	2
1.2	Attenuator WEC device: Pelamis [Pelamis Wave Power, UK] [2] . . . . .	3
1.3	Terminator WEC device: Oyster [Aquamarine Power, UK] [3] . . . . .	4
1.4	Point absorbers as an omnidirectional devices . . . . .	4
1.5	Antenna effect of point absorber devices . . . . .	4
1.6	Point absorber WEC device: Powerbuoy [Ocean Power Technologies, USA] [4] . . . . .	5
1.7	Different PTO mechanisms [5] . . . . .	6
1.8	Typical hydraulic circuit of WEC [5] . . . . .	6
1.9	A schematic for a linear generator based on a permanent magnets [5] . . . .	7
1.10	Displacement of a heaving buoy WEC with latching control . . . . .	10
1.11	Layout of bottom-hinged pitching point absorber experimental setup . . . .	11
1.12	Schematic for the tested OWC . . . . .	13
1.13	Schematic for a PTO system with a hydraulic circuit . . . . .	14
2.1	P-M Energy spectral density $S(\omega)$ at different wind speeds $U$ . . . . .	19
2.2	Peak frequency $\omega_m$ and Energy frequency $\omega_e$ at different wind speeds $U$ . .	19
2.3	Significant wave height $H_s$ at different wind speeds $U$ . . . . .	20
2.4	Available wave power $P$ at different wind speeds $U$ . . . . .	20
2.5	Schematic representation of cylindrical heaving buoy WEC . . . . .	21
3.1	Schematic representation of the control system installed on the heaving buoy	30
3.2	Schematic representation of the CVS sheaves . . . . .	30
3.3	V-belt drive running diameters $D_\alpha$ and sheaves axial position $S_\alpha$ for the CVS system in Table 3.1 . . . . .	33
3.4	Controller bandwidth . . . . .	35
3.5	Wave energy frequency $\omega_e$ vs. wind speed $U$ according to PM-spectrum calculations . . . . .	36
4.1	Wave Excitation force amplitude $ \hat{F}_e $ at different wind speeds $U$ . . . . .	39
4.2	Amplification ratio $AR$ at different PTO damping $b$ in regular waves . . . .	40
4.3	Buoy velocity amplitude $\dot{z}$ at different PTO damping $b$ in regular waves . . .	41
4.4	Average absorbed power $P_{av}$ at different PTO damping $b$ in regular waves .	41
4.5	Efficiency $\eta$ and capture width $\lambda$ at different PTO damping $b$ in regular waves	42
4.6	Amplification ratio $AR$ at different PTO damping $b$ in irregular waves . . . .	43
4.7	Buoy velocity amplitude $\dot{z}$ in RMS at different PTO damping $b$ in irregular waves . . . . .	44
4.8	Average absorbed power $P_{av}$ in RMS at different PTO damping $b$ in irregular waves . . . . .	44
4.9	Efficiency $\eta$ and capture width $\lambda$ in RMS at different PTO damping $b$ in irregular waves . . . . .	45
4.10	Schematic representation of the experimental model . . . . .	46
4.11	The experimental setup CAD model . . . . .	47
4.12	The experimental setup . . . . .	47

4.13	The exciter mechanism CAD model . . . . .	48
4.14	The controller mechanism CAD model . . . . .	49
4.15	The controller mechanism . . . . .	49
4.16	The sheaves shaft CAD model . . . . .	50
4.17	The speed ratio $\Delta$ at different input excitation frequencies $\omega_e$ . . . . .	50
4.18	Excitation input for Setting 1 at 15.71 rad/s . . . . .	52
4.19	Displacement amplitude at different input excitation frequencies (Setting 1: 6.16 rad/s, Setting 2: 6.83 rad/s) . . . . .	53
4.20	Output phase at different input excitation frequencies (Setting 1: 6.16 rad/s, Setting 2: 6.83 rad/s) . . . . .	53
4.21	Experimental steady state measurements for Setting 1 at 6.16 rad/s . . . . .	54
4.22	Experimental steady state measurements for Setting 1 at 6.83 rad/s . . . . .	54
C.1	The deterministic quantity $y(t)$ magnitude at wind speed $U$ of 10 m/s . . . . .	108
C.2	The deterministic quantity $y(t)$ real part at wind speed $U$ of 10 m/s . . . . .	109
C.3	The deterministic quantity $y(t)$ imaginary part at wind speed $U$ of 10 m/s . . . . .	109
C.4	The DFT of the deterministic quantity $y(t)$ magnitude at wind speed $U$ of 10 m/s . . . . .	110
C.5	DFT for the time domain signal calculated by the irregular simulation method Eqn.(2.15) at wind speed $U$ of 10 m/s . . . . .	111
C.6	Buoy displacement at wind speed $U$ of 10 m/s at PTO damping $b$ of 0.5 kN.s/m	111
C.7	Buoy displacement at wind speed $U$ of 10 m/s at PTO damping $b$ of 1 kN.s/m	112
C.8	Buoy displacement at wind speed $U$ of 10 m/s at PTO damping $b$ of 2 kN.s/m	112
C.9	Buoy displacement at wind speed $U$ of 10 m/s at PTO damping $b$ of 5 kN.s/m	113
C.10	Buoy displacement at wind speed $U$ of 10 m/s at PTO damping $b$ of 10 kN.s/m	113
C.11	Amplification ratio $AR$ at different PTO damping $b$ and wind speeds $U$ . . . . .	114
C.12	Displacement difference at different PTO damping $b$ and wind speeds $U$ . . . . .	114

# Nomenclature

$\Delta$	Speed ratio
$\delta$	Kronecker delta function
$\eta$	Power absorption efficiency
$\kappa$	Convolution kernel
$\lambda$	Capture width
$\lambda_{rd}$	Multiplication factor with inverse time dimensions
$\rho$	Density of sea water
$\phi$	Taper angle of the sheave
$\phi_i$	Random phase for irregular waves modeling
$\psi_e(t)$	Even function of the impulse response function for radiation
$\psi_o(t)$	Odd function of the impulse response function for radiation
$\psi_r(t)$	Impulse response function for radiation
$\omega$	Frequency in (rad/s)
$\omega_e$	Wave energy frequency in (rad/s)
$\omega_m$	Peak frequency in (rad/s)
$\omega_n$	System natural frequency in (rad/s)
$\omega_{nb}$	Heaving buoy natural frequency in (rad/s)
$\omega_z$	Zero-up crossing wave frequency in (rad/s)
$a(\omega)$	Acceleration in frequency domain
$\hat{a}$	Acceleration in complex amplitude
A	Bretschneider formula parameter dependent on the maximum frequency
AR	Amplification ratio
$A_b$	Heaving buoy cross-sectional area crossing the water surface
$A_{ps}$	System matrix for the radiation force state space sub-system
$A_s$	System matrix of the state space model
$b$	Power take-off system damping

$b_{opt}$	Power take-off system optimum damping
$b_r$	hydrodynamic damping
B	Bretschneider formula parameter dependent on the root mean square value of the spectrum
$B_{ps}$	System vector for the radiation force state space sub-system
$B_s$	System vector of the state space model
$C_p$	Central distance between pulleys
$C_{ps}$	System vector for the radiation force state space sub-system
$C_s$	System vector of the state space model
$d$	Heaving buoy draft (submerged height)
$D_\alpha$	Running diameter of pulley $\alpha$
$D_{\alpha,min}$	Minimum running diameter for sheave $\alpha$
$D_b$	Heaving buoy diameter
$D_{pb}$	Buoy-side pulley running diameter
$D_{ps}$	External-spring-side pulley running diameter
E	Energy spectral density function
$f_d$	Damping force
$f_e$	Excitation force
$f_r$	Radiation force
$f_s$	Restoring force
$f_t$	Tuning force
$\hat{F}_e$	Heave excitation force complex amplitude
$\hat{F}_r$	Radiation force complex amplitude
$g$	Gravitational acceleration
$h$	Water depth
$H_0^{(1)}$	Hankel function of first kind of order 0
$\hat{H}_0^{(1)}$	First derivative of Hankel function of first kind of order 0
$H_{1/3}$	Average wave height of the highest one-third waves

$H_s$	Significant wave height
$i$	Complex number
$I_0$	The first derivative of the modified Bessel function of the first kind of order 0
$\dot{I}_0$	Modified Bessel function of the first kind of order 0
$I_1$	Modified Bessel function of the first kind of order 1
$I_p$	Moment of inertia of the pulleys
$J_0$	First Bessel function of order zero
$J'_0$	First derivative of first Bessel function of order zero
$k$	Stiffness of the system
$k_0$	Modified Bessel function of the second kind of order 0
$\dot{k}_0$	First derivative of modified Bessel function of the second kind of order 0
$k_b$	Heaving buoy buoyancy stiffness
$k_{controller}$	Controller variable external stiffness
$k_{eff}$	System effective stiffness
$k_{ext}$	External stiffness
$L_p$	V-belt pitch length
$m_0$	Zeroth spectral moment
$m_2$	Second spectral moment
$m_b$	Heaving buoy mass
$m_{eq}$	Equivalent oscillating mass of the moving components of the controller
$m_n$	$n_{th}$ spectral moment
$m_r$	hydrodynamic added mass
$m_{tot}$	Total oscillating mass including the controller inertia effect
$P$	Available power per unit width of wave front (power flux)
$P_{av}$	Average absorbed power
$P_k$	Kinetic power
$P_m$	Power consumed in the mechanical damper
$P_{m,max}$	Maximum power consumed in the mechanical damper