

AIN SHAMS UNIVERSITY FACULTY OF ENGINEERING STRUCTURAL ENGINEERING DEPARTMENT

EFFECT OF HANGER STEEL REINFORCEMENT AND ITS LOCATION ON THE BEHAVIOUR OF LEDGE BEAM

A Thesis Submitted in Partial Fulfillment of the Requirements for the Degree of

DOCTOR OF PHILOSOPHY
in
CIVIL ENGINEERING (STRUCTURES)

by

EZZ EL-DEEN MOSTAFA SALAH EL-DEEN ARAFA

Supervised by

Prof. Osama Hamdy Abd El-Wahed

Professor of Reinforced Concrete Structures Ain Shams University

Prof. Fathy A. Saad

Professor of Reinforced Concrete Structures Ain Shams University

Cairo - 2015

Researcher name:

Ezz El-Deen Mostafa Salah El-Deen Arafa

Thesis Title:

EFFECT OF HANGER STEEL REINFORCEMENT AND ITS LOCATION ON THE BEHAVIOUR OF LEDGE BEAM

Referees committee:

1- Prof. Balthasar Novak

Professor of Reinforced Concrete Structures

Faculty of Engineering – Stuttgart University

2- Prof. Amr Ali Abdelrahman

Professor of Reinforced Concrete Structures

Faculty of Engineering – Ain Shams University

3- Prof. Fathy A.Saad

Professor of Reinforced Concrete Structures

Faculty of Engineering – Ain Shams University

4- Prof. Osama Hamdy Abd El-Wahed

Professor of Reinforced Concrete Structures

Faculty of Engineering – Ain Shams University

STATEMENT

This thesis is submitted as partial fulfillment of the requirements for the

degree of Doctor of Philosophy in Civil Engineering (Structures),

Faculty of Engineering, Ain Shams University.

The author carried out the work included in this thesis, and no part of it

has been submitted for a degree or a qualification at any other scientific

entity.

Date : December 2015

Name : Ezz El-Deen Mostafa Salah El-Deen Arafa

Signature :

AUTHOR

Name : Ezz El-Deen Mostafa Salah El-Deen Arafa

Date of birth : 24 June 1979

Place of birth : Saudi Arabia

Academic Degree: M.Sc. in Civil Engineering

Major : Structural Engineering

University : Ain Shams University

Date : February 2008

Academic Degree: B.Sc. in Civil Engineering

Major : Structural Engineering

University : Ain Shams University

Date : June 2001

Grade : Distinction with Honor

Current job : Teaching and Research Assistant

Faculty of Engineering — Ain Shams University

ACKNOWLEDGEMENT

All praise belongs to God, Most Gracious, and Most Merciful. Only through his grace and guidance, this work was completed.

I would like to express my deepest gratitude to **Prof. Fathy A.Saad,** Professor of Reinforced concrete structures, Ain Shams University, for his constant supervision, planning, guidance and valuable suggestions during all the phases of this work. I'm also extremely thankful to **Prof. Osama Hamdy Abd El-Wahed**, Professor of Reinforced Concrete Structures, Ain Shams University, for his guidance and valuable suggestions.

I would like to thank **Prof. Amr Ali Abdelrahman**, Professor of Reinforced concrete structures, Ain Shams University for his support and valuable suggestions which helped me in the accomplishment of this work.

Finally, I am particularly grateful to my mother and my brother for helping and assisting me in all the stages of this work.

The work is dedicated to the soul of my father.

EFFECT OF HANGER STEEL REINFORCEMENT AND ITS LOCATION ON THE BEHAVIOUR OF LEDGE BEAM

SUMMARY

Reinforced concrete L-shaped beams are frequently used in the precast concrete industry especially in bridge constructions to support a series of deck beams. The most common application is where L-shaped beams are used to support double-tee deck beams in a parking garage. The stems of the double-tee deck beams bear on the top surface of the spandrel ledge, loading the spandrel beam with a series of concentrated eccentric reactions. The contribution of the inner vertical branches of stirrups as a hanger for the ledge part is neglected. Therefore, the outer vertical stirrups have a great amount with respect to the internal stirrups.

This research introduces experimental and analytical study in order to investigate the load path of the concentrated loads on the ledge beams and study the effect of the inner vertical stirrups in the contribution of the hanging action of ledge part. The objective of this thesis is to find an efficient, easy and economic reinforcement arrangement under combined effect of bending moment, shear force and torsional moment.

The experimental program is designed to study the contribution of the internal vertical stirrups on the hanging action of the ledge. The test specimens consist of sixteen simply supported beams of 2.70m span subdivided into six groups. All beams had a total height of 380mm and a web width of 120mm or 250mm. The heights of the ledge part were chosen to be 140, 180 & 220mm and their projection was 250mm. The study considers the following variables, the distribution of internal vertical stirrups, thickness of ledge part, the web width of specimens and eccentricity of vertical loads. The specimens were designed to ensure that the ultimate failure load of the ledge part is due to yielding of the vertical hanger outer stirrups based on the ACI assumptions. The specimens were tested under single vertical concentrated load at mid-span acting on the ledge part with eccentricity of 150, 100 & 70mm to the inner edge of the web.

An additional study using finite element method has been carried out to introduce a design model for the ledge beams. The commercial finite element program Ansys has been adapted in which the experimental tests has been analyzed. Additional parametric study can be done to investigate more variables. A verification of the adapted model has been run the experimental tested specimens.

An analysis using strut and tie method has been performed. Different strut and tie models had been built to simulate the behavior of the ledge beams and can be easily used into design.

Key Words: Shear friction, Shear reinforcement, Ledge beam.

To fulfill the previously mentioned objectives, this research is divided into the following chapters:

- Chapter (1) is an introduction to this research. This chapter discusses the importance of the research and the scope of the research program.
- Chapter (2) presents a brief review of the available literature, research findings, performance data and current practices relative to the behavior and design of reinforced concrete ledge beams.
- Chapter (3) presents the experimental program. This chapter describes the formwork, material properties, fabrication, test specimens, test setup, measurement devices, and loading scheme.
- Chapter (4) discusses and analyses the test results and observations. The results include load–deflection and load-rotation curves, crack patterns and strain in both steel and concrete.
- Chapter (5) discusses the methodology adopted in the analytical study along with the description of the analysis technique adopted by the finite element computer program (ANSYS) used and presents details of the analyzed models, material properties and main analysis results. Also, a parametric study had been

established to take other variables which are not taken in the experimental program.

- Chapter (6) introduces analytical study using the strut and tie method. The methodology of the model creation been introduced in details. Different strut and tie models have been constructed for different tested specimens that simulate behavior. Based on these strut and tie models, the failure loads of the tested specimens have been predicted. These models have been verified. Finally, a rational design tool using strut and tie method for those ledge beams has been conducted, that can be easily used by the designer of those elements.
- Chapter (7) includes the general conclusions of the study alongside with recommendations for future research and developments in the subject.

TABLE OF CONTENTS	
TABLE OF CONTENTS	-i-
LIST OF FIGURES	-iv-
LIST OF TABLES	-X-
Abstract	-xi-
Chapter 1 INTRODUCTION	-1-
1.1 Background	-1-
1.2 Objectives	-2-
1.3 Contents of the Research	-2-
Chapter 2 LITERATURE REVIEW	-4-
2.1 Introduction	-5-
2.2 Background	-6-
2.3 Previous Research	-7-
2.4 Lateral Restrains with Deck-Tie Plates	-9-
2.5 Laterally Unrestrained and Concentrically Loaded Beams	-11-
2.6 Laterally Unrestrained and Eccentrically Loaded Beams	-15-
2.7 Laterally Restrained and Eccentrically Loaded Beams	-19-
2.8 Bending Moment -Lateral Deflection Curves	-22-
2.9 Failure Modes of Ledges	-26-
2.10 Current Practice	-28-
2.10.a PCI Design Handbook	-28-
2.10.b PCA Notes on ACI 318-14	-32-
2.11 Previous Studies	-33-
Chapter 3 EXPERIMENTAL PROGRAM	-44-
3.1 Introduction	-44-
3.2 Experimental Program	-44-
3.3 Test Setup and Loading Scheme	-45-
3.4 Test Specimens	-47-
3.4.1 Concrete Dimensions of Beams	-47-
3.4.2 Design and Labeling of Test Specimens	-48-
3.5 Material Properties	-57-
3.5.1 Concrete	-57-
3.5.2 Compressive Strength	-58-
3.5.3 Steel Reinforcement	-59-
3.6 Fabrication of Test Specimens	-61-
3.7 Instrumentation	-61-
3.7.1 Deflectometer Locations of Test Specimens	-61-
3.7.2 Strain Gauges	-65-
3.7.3 Data Acquisition	-71-
3.8 Testing Procedure and Preparation for Testing	-71-
Chapter 4 ANALYSIS AND DISCUSSION OF EXPERIMENTAL	
RESULTS	-73-
4.1 Introduction	-73-
4.2 Modes of Failure for the Tested Specimens	-73-
4.2.1 Crack Initiation and Propagation	-73-
4.2.2 Crack Pattern and Failure Mode	-74-
4.2.2.1 Group (I)	-75-
4.2.2.2 Group (II)	-81-
4.2.2.3 Group (III)	-85-
4.2.2.4 Group (IV)	-89-
4.2.2.5 Group (V)	-93-
4.2.2.6 Group (VI)	-96-
L / /	

4.3 Deformations and Rotations of Tested Specimens	-98-
4.3.1 Group (I)	-98-
4.3.2 Group (II)	-100-
4.3.3 Group (III)	-101-
4.3.4 Group (IV)	-103-
4.3.5 Group (V)	-104-
4.3.6 Group (VI)	-106-
4.4 Strain of Steel Stirrups of Tested Specimens	-107-
4.4.1 Group (I)	-107-
4.4.2 Group (II)	-110-
4.4.3 Group (III)	-112-
4.4.4 Group (IV)	-114-
4.4.5 Group (V)	-115-
4.4.6 Group (VI)	-116-
4.5 Strain of Top Compressed Concrete of Tested Specimens	-118-
4.5.1 Group (I)	-118-
4.5.2 Group (II)	-119-
4.5.3 Group (III)	-119-
4.5.4 Group (IV)	-120-
4.5.5 Group (V)	-121-
4.5.6 Group (VI)	-122-
Chapter 5 FINITE ELEMENT ANALYSIS	-124-
5.1 General	-124-
5.2 Formulation	-126-
5.3 ANSYS Overview	-126-
5.3.1 Basic Analysis Procedure of ANSYS	-127-
5.3.2 Structural Analysis of ANSYS	-130-
5.3.2.1 Nonlinear Analysis in Concrete Structures	-132-
5.3.2.2 Static Analysis	-135-
5.4 Modeling Approach of Concrete Ledge Beams	-136-
5.4.1 Generation of the Finite Element Model	-137-
5.5 Verification of the Analytical Results	-142-
5.5.1 Cracking and Failure Loads	-142-
5.5.1.1 Group (I)	-145-
5.5.1.2 Group (II)	-147-
5.5.1.3 Group (III)	-149-
5.5.1.4 Group (IV)	-151-
5.5.1.5 Group (V)	-151-
5.5.1.6 Group (VI)	-153-
5.5.2 Deflections and Strains of Tested Specimens	-154-
5.5.2.1 Group (I)	-154-
5.5.2.2 Group (II)	-156-
5.5.2.3 Group (III)	-157-
5.5.2.4 Group (IV)	-159-
5.5.2.5 Group (V)	-160-
5.5.2.6 Group (VI)	-162-
5.5.3 Parametric Study	-163-
5.5.3.1 Cracking and Failure Loads for Specimen (A-4)	-164-
5.5.3.2 Deflections and Strains of Specimen (A-4)	-165-
Chapter 6 STRUT AND TIE ANALYSIS	-167-
6.1 General	-167-
6.2 Background	-168-

6.2.1 Previous Studies for the Applications of STM for Reinforced Concrete Ledge Beams	-171-
6.3 Analysis of Tested Specimens	-173-
6.3.1 Description of Load Pat	-173-
6.3.2 Proposal of Strut and Tie Model for Ledge Beams	-174-
6.3.3 Design for Ledge Beams Using Strut and Tie Model	-179-
6.4 Application of Proposal Strut and Tie Model for Tested Specimens	-181-
6.4.1 Specimen (A-1)	-181-
6.4.1.1 Dimensioning of the Struts and Ties for the 3-D Model of specimen	-185-
(A-1)	
6.4.1.2 Analysis of Strut and Tie Model for Specimen (A-1)	-187-
6.4.1.3 The Average Tensile Stress in the Vertical Stirrups and the Tensile	
Stress at Midspan in the Bottom Longitudinal Reinforcement for Specimen	-188-
(A-1)	
6.4.2 Specimen (A-1-7)	-190-
6.4.2.1 Dimensioning of the Struts and Ties for the 3-D Model of specimen (A-1-7)	-193-
6.4.2.2 Analysis of Strut and Tie Model for Specimen (A-1-7)	-195-
6.4.2.3 The Average Tensile Stress in the Vertical Stirrups and the	170
Tensile Stress at Midspan in the Bottom Longitudinal Reinforcement for	-196-
Specimen (A-1-7)	170
6.4.3 Specimen (B-1)	-198-
6.4.3.1 Dimensioning of the Struts and Ties for the 3-D Model of specimen	
(B-1)	-201-
6.4.3.2 Analysis of Strut and Tie Model for Specimen (B-1)	-203-
6.4.3.3 The Average Tensile Stress in the Vertical Stirrups and the	
Tensile Stress at Midspan in the Bottom Longitudinal Reinforcement for	-204-
Specimen (B-1)	
6.4.4 Specimen (C-1)	-206-
6.4.4.1 Dimensioning of the Struts and Ties for the 3-D Model of specimen	200
(C-1)	-209-
6.4.4.2 Analysis of Strut and Tie Model for Specimen (C-1)	-211-
6.4.4.3 The Average Tensile Stress in the Vertical Stirrups and the	
Tensile Stress at Midspan in the Bottom Longitudinal Reinforcement for	-212-
Specimen (C-1)	
6.4.5 Specimen (C-1-7)	-214-
6.4.5.1 Dimensioning of the Struts and Ties for the 3-D Model of	-217-
specimen (C-1-7)	-21/-
6.4.5.2 Analysis of Strut and Tie Model for Specimen (C-1-7)	-219-
6.4.5.3 The Average Tensile Stress in the Vertical Stirrups and the	
Tensile Stress at Midspan in the Bottom Longitudinal Reinforcement for	-221-
Specimen (C-1-7)	
6.5 Summary of the STM Verification and Outcomes	-223-
Chapter 7 SUMMARY AND CONCLUSIONS	-226-
7.1 Summary	-226-
7.2 Conclusions	-226-
7.3 Suggestions for Future STUDIES	-228-
REFRENCES	-230-
	-

LIST OF FIGURES	
Chapter 2	
Figure (2-1): Precast and Prestressed Concrete Spandrel Beams	-7-
Figure (2-2): The Spandrel to Double Tee Connection	-8-
Figure (2-3): Deformations of Laterally Restrained and Unrestrained Beams	-11-
Figure (2-4): (a): Global and Local Coordinate Systems and (b): Sign	
Conventions for Positive Internal Moments	-13-
Figure (2-5): Deformation In x-y, y-z And z-x	-13-
Figure (2-6): Components of Bending Moment Mx along Local Axes x*, y*& z*	-13-
Figure (2-7): (a): Laterally Unrestrained Beam under Eccentric Loading and (b): Components of Moment Mz in the Local Axes	-13-
Figure (2-8): Actual and Equivalent Loading for Laterally Unrestrained	1.5
Beam	-17-
Figure (2-9): (a): Laterally Restrained Beam under Eccentric Loading and	20
(b): Components of Moment My in the Local Axes	-20-
Figure (2-10): Bending Moment-Maximum Lateral Deflection Curves for Rectangular Beams with Various Values of Eccentricity (h/L=0.11	-24-
Figure (2-11): Lateral Deflections at the Top and Centroid of the Laterally Unrestrained Rectangular Beams (h/L=0.11)	-25-
Figure (2-13): Failure Surfaces At End and Inner Locations (Rath, 1984)	-27-
Figure (2-14): PCI Simplified Failure Surfaces at End and Inner Locations	-27-
Figure (2-15): Independent Failure	-29-
Figure (2-16): Overlapped Failure	-29-
Figure (2-17): Design of Transverse Bending of Ledge (PCI Design Handbook, 2010)	-30-
Figure (2-18): Ledge Hanger Steel Geometry (PCI Design Handbook, 2010)	-31-
Figure (2-19): Hanger Reinforcement to Prevent Separation of Ledge from Stem (PCA Notes on ACI 318-14)	-33-
Figure (2-20): Comparison between Failure Surfaces Used By PCI (2010) and Rath (1984)	-36-
Figure (2-21): Punching Shear Failures in the Second Specimen (Klein, 1986a)	-37-
Figure (2-22): Distribution of Principle Compressive Strains at Failure (Hassan, 2007)	-38-
Figure (2-23): Failure of a Ledge with Embedded Bearing Plates, SP1	-40-
Figure (2-24): Spandrel Ledge after Punching Shear Failure, SP14	-41-
Figure (2-25): Spandrel Ledge after Punching Shear Failure, SP15	-41-
Figure (2-26): View of the Second Reteset, SP16	-42-
Figure (2-27): Intersection of Road El-Farag Axis with Cairo-Alexandria	
Desert Road (km39)-Egypt (Under Construction)	-43-
Chapter 3	
Figure (3-1): Preparation of Formwork	-45-
Figure (3-2): Loading Scheme for Tested Specimens	-45-
Figure (3-3a): Figure (3-3a): Vertical Supports for tested specimens	-46-
Figure (3-3b): Lateral Supports for Tested Specimens	-46-
Figure (3-3c): Hydraulic Jack 300 kN Capacity	-46-
Figure (3-4): General Concrete Dimensions of Tested Specimens	-48-

Figure (3-5a): Concrete Dimensions and Reinforcement Details for	-49-
Specimen (A-1) of Group (I)	
Figure (3-5b): Concrete Dimensions and Reinforcement Details for	5 0
Specimen (A-2) of Group (I)	-50-
Figure (3-6a): Concrete Dimensions and Reinforcement Details for	-51-
Specimen (B-1) of Group (II)	<i>J</i> 1
Figure (3-6b): Concrete Dimensions and Reinforcement Details for	-52-
Specimen (B-2) of Group (II)	-32-
Figure (3-6c): Concrete Dimensions and Reinforcement Details for	-52-
Specimen (B-3) of Group (II)	-32-
Figure (3-7a): Concrete Dimensions and Reinforcement Details for	52
Specimen (C-1) of Group (III)	-53-
Figure (3-7b): Concrete Dimensions and Reinforcement Details for	<i>5</i> 4
Specimen (C-2) of Group (III)	-54-
Figure (3-7c): Concrete Dimensions and Reinforcement Details for	- 4
Specimen (C-3) of Group (III)	-54-
Figure (3-8a): Concrete Dimensions and Reinforcement Details for	
Specimen (A-0) of Group (IV)	-55-
Figure (3-8b): Concrete Dimensions and Reinforcement Details for	
Specimen (B-0) of Group (IV)	-56-
Figure (3-8c): Concrete Dimensions and Reinforcement Details for	-56-
Specimen (C-0) of Group (IV)	50
Figure (3-9): Stress Strain Curve for the Used Mild Steel	-59-
Figure (3-10): Stress Strain Curve for the Used High Tensile Steel	-60-
Figure (3-11): Preparation of Formwork	-61-
Figure (3-12a):Locations of Deflectometers for Specimens (A-0,B-0 & C-0)	-63-
Figure (3-12b): Locations of Deflectometers for Other Specimen	-64-
Figure (3-13a): Locations of Steel Strain Gauges for Group (I&V)	-66-
Figure (3-13b): Locations of Steel Strain Gauges for Group (II	-67-
Figure (3-13c): Locations of Steel Strain Gauges for Group (III&VI)	-68-
Figure (3-13d): Locations of Steel Strain Gauges for Group (IV)	-69-
Figure (3-14): Locations of Concrete Strain Gauges for All Specimens	-70-
Figure (3-15): Data Acquisition Device and Channel Box	-71-
Figure (3-16): Load Eccentricity for Tested Specimens	-72-
Chapter 4	, _
Figure (4-1a): Crack Pattern and Failure Shape for Specimen (A-1) of	
Group (I)	70
Group (1)	-78-
	-/8-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of	-79-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I)	
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of	
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I)	-79-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of	-79- -80-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II)	-79-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of	-79- -80- -82-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II)	-79- -80-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of	-79- -80- -82- -83-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II)	-79- -80- -82-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II) Figure (4-2c): Crack Pattern and Failure Shape for Specimen (B-3) of	-79- -80- -82- -83-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II) Figure (4-2c): Crack Pattern and Failure Shape for Specimen (B-3) of Group (II)	-79- -80- -82- -83-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II) Figure (4-2c): Crack Pattern and Failure Shape for Specimen (B-3) of Group (II) Figure (4-3a): Crack Pattern and Failure Shape for Specimen (C-1) of	-79- -80- -82- -83- -84-
Figure (4-1b): Crack Pattern and Failure Shape for Specimen (A-2) of Group (I) Figure (4-1c): Crack Pattern and Failure Shape for Specimen (A-3) of Group (I) Figure (4-2a): Crack Pattern and Failure Shape for Specimen (B-1) of Group (II) Figure (4-2b): Crack Pattern and Failure Shape for Specimen (B-2) of Group (II) Figure (4-2c): Crack Pattern and Failure Shape for Specimen (B-3) of Group (II) Figure (4-3a): Crack Pattern and Failure Shape for Specimen (C-1) of Group (III)	-79- -80- -82- -83- -84-

Figure (4-3c): Crack Pattern and Failure Shape for Specimen (C-3) of Group (III)	-88-
± · · /	
Figure (4-4a): Crack Pattern and Failure Shape for Specimen (A-0) of Group (IV)	-90-
Figure (4-4b): Crack Pattern and Failure Shape for Specimen (B-0) of	
Group (IV)	-91-
Figure (4-4c): Crack Pattern and Failure Shape for Specimen (C-0) of	
	-92-
Group (IV) Figure (4.5a): Creek Bettern and Failure Shape for Specimen (4.1.7) of	
Figure (4-5a): Crack Pattern and Failure Shape for Specimen (A-1-7) of Group (V)	-94-
Figure (4-5b): Crack Pattern and Failure Shape for Specimen (A-1-10) of	-95-
Group (V) Figure (4.6a): Creak Pattern and Failure Shape for Specimen (C. 1.7) of	
Figure (4-6a): Crack Pattern and Failure Shape for Specimen (C-1-7) of	-97-
Group (VI) Figure (4.6b): Greek Pettern and Feilure Shape for Specimen (C. 1.10) of	
Figure (4-6b): Crack Pattern and Failure Shape for Specimen (C-1-10) of	-97-
Group (VI) Figure (4.7a): Load Deflection Courses for Cross (I)	00
Figure (4-7a): Load Deflection Curves for Group (I)	<u>-99-</u>
Figure (4-7b): Load Rotation Curves for Group (I)	<u>-99-</u>
Figure (4-8a): Load Deflection Curves for Group (II)	-100-
Figure (4-8b): Load Rotation Curves for Group (II)	-101-
Figure (4-9a): Load Deflection Curves for Group (III)	-102-
Figure (4-9b): Load Rotation Curves for Group (III)	-102-
Figure (4-10a): Load Deflection Curves for Group (IV)	-103-
Figure (4-10b): Load Rotation Curves for Group (IV)	-104-
Figure (4-11a): Load Deflection Curves for Group (I)	-105-
Figure (4-11b): Load Rotation Curves for Group (I)	-105-
Figure (4-12a): Load Deflection Curves for Group (VI)	-106-
Figure (4-12b): Load Rotation Curves for Group (VI)	-107-
Figure (4-13a): Load Strain Curves for Midspan External Stirrups of Group	-108-
(I)	100
Figure (4-13b): Load Strain Curves for Midspan Additional External	-109-
Stirrups of Group (I)	-10)-
Figure (4-13b): Load Strain Curves for Midspan Additional External	-109-
Stirrups of Group (I)	-10)-
Figure (4-13c): Load Strain Curves for Midspan Internal Towards Ledge	-109-
Stirrups of Group (I)	-10)-
Figure (4-14a): Load Strain Curves for Midspan External Stirrups of Group	-110-
(II)	-110-
Figure (4-14b): Load Strain Curves for Midspan Additional External	-111-
Stirrups of Group (II)	-111-
Figure (4-14c): Load Strain Curves for Midspan Internal Towards Ledge	-111-
Stirrups of Group (II)	-111-
Figure (4-15a): Load Strain Curves for Midspan External Stirrups of Group	112
(III)	-112-
Figure (4-15b): Load Strain Curves for Midspan Additional External	112
` '	-113-
Figure (4-15b): Load Strain Curves for Midspan Additional External	
Figure (4-15b): Load Strain Curves for Midspan Additional External Stirrups of Group (III)	-113- -113-
Figure (4-15b): Load Strain Curves for Midspan Additional External Stirrups of Group (III) Figure (4-15c): Load Strain Curves for Midspan Internal Towards Ledge	-113-
Figure (4-15b): Load Strain Curves for Midspan Additional External Stirrups of Group (III) Figure (4-15c): Load Strain Curves for Midspan Internal Towards Ledge Stirrups of Group (III)	
Figure (4-15b): Load Strain Curves for Midspan Additional External Stirrups of Group (III) Figure (4-15c): Load Strain Curves for Midspan Internal Towards Ledge Stirrups of Group (III) Figure (4-16): Load Strain Curves for Midspan External Stirrups of Group	-113- -114-
Figure (4-15b): Load Strain Curves for Midspan Additional External Stirrups of Group (III) Figure (4-15c): Load Strain Curves for Midspan Internal Towards Ledge Stirrups of Group (III) Figure (4-16): Load Strain Curves for Midspan External Stirrups of Group (IV)	-113-

Figure (4-17b): Load Strain Curves for Midspan Additional External Stirrups of Group (V)	-116-
Figure (4-18a): Load Strain Curves for Midspan External Stirrups of Group (VI)	-117-
Figure (4-18b): Load Strain Curves for Midspan Additional External	
Stirrups of Group (VI)	-117-
Figure (4-19): Load Strain Curves for Concrete at Midspan Top	
Compressed Surface of Group (I)	-118-
Figure (4-20): Load Strain Curves for Concrete at Midspan Top	
Compressed Surface of Group (II)	-119-
Figure (4-21): Load Strain Curves for Concrete at Midspan Top	-120-
Compressed Surface of Group (III)	120
Figure (4-22): Load Strain Curves for Concrete at Midspan Top	-121-
Compressed Surface of Group (IV)	121
Figure (4-23): Load Strain Curves for Concrete at Midspan Top	-122-
Compressed Surface of Group (V)	-122-
Figure (4-24): Load Strain Curves for Concrete at Midspan Top	-123-
Compressed Surface of Group (VI)	-123-
Chapter 5	
Figure (5-1): Stress Strain Curve of Concrete	-138-
Figure (5-2a): Solid65 (ANSYS Version 14, 2012)	-140-
Figure (5-2b): Cracking Plane in Solid65 (ANSYS 1998)	-141-
Figure (5-3): The ANSYS Finite Element Type Link 180 (3D spar)	-142-
Figure (5-4): Experimental Failure Loads versus Analytical F.E. Failure	
Loads for Tested Specimen	-144-
Figure (5-5a): ANSYS F.E. Model of Specimen (A-1)	-145-
Figure (5-5b): ANSYS F.E. Model of Specimen (A-2)	-146-
Figure (5-5c): ANSYS F.E. Model of Specimen (A-3)	-146-
Figure (5-6): ANSYS Failure Mode for groups (I, II, V)	-147-
Figure (5-7a): ANSYS F.E. Model of Specimen (B-1)	-148-
Figure (5-7b): ANSYS F.E. Model of Specimen (B-2)	-148-
Figure (5-7c): ANSYS F.E. Model of Specimen (B-3)	-149-
Figure (5-8a): ANSYS F.E. Model of Specimen (C-1)	-150-
Figure (5-8b): ANSYS F.E. Model of Specimen (C-2)	-150-
Figure (5-8c): ANSYS F.E. Model of Specimen (C-3)	-151-
Figure (5-9): ANSYS Failure Mode for groups (III & VI) and Specimen	-151-
(C-0)	
Figure (5-10): ANSYS F.E. Model of Specimens (A-0, B-0 & C-0)	-152-
Figure (5-11): ANSYS Failure Mode for Specimens (A-0 & B-0)	-153-
Figure (5-12a): Load Deflection Curves for Analytical F.E. Group (I)	-155-
Figure (5-12b): Load Strain Curves for Midspan External Stirrups of	-155-
Analytical F.E. Group (I)	
Figure (5-13a): Load Deflection Curves for Analytical F.E. Group (II)	-156-
Figure (5-13b): Load Strain Curves for Midspan External Stirrups of	-157-
Analytical F.E. Group (II) Figure (5.14a): Lead Deflection Courses for Analytical F.E. Group (III)	150
Figure (5-14a): Load Deflection Curves for Analytical F.E. Group (III)	-158-
Figure (5-14b): Load Strain Curves for Midspan External Stirrups of Analytical F.E. Group (III)	-158-
Figure (5-15a): Load Deflection Curves for Analytical F.E. Group (IV)	-159-
Figure (5-15b): Load Strain Curves for Midspan External Stirrups of	
Analytical F.E. Group (IV)	-160-