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Faculty of Science
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Folding of Geometric Figures and Some Applications in Algebra

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(Pure Mathematics)

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Summary

Summary

This thesis sheds light on the concept of folding for some geometric figures, KU-graph of KU-algebra and KU-ideal of KU-algebra.

The folding of a manifold is a one of the famous problems in the fields of geometry and topology. S.A.Robertson in [73] is the first one who introduced this idea, when he crumpled a sheet of paper in his hand and then crushed it flat against a desk-top and studied the stratification determined by the folds. Most folding and unfolding problems are attractive from a pure mathematical standpoint for the beauty of the problems themselves. However, most of the problems have close connections to important industrial applications. Paper folding has applications in sheet metal bending, packaging, and airbag folding. Unfolding polyhedra has applications in manufacturing, particularly sheet metal bending.

Folding of graphs began with M. El-Ghouls work, in [22]. The conditional folding of manifold and a graph folding have been defined by El-Kholy [53, 54]. Unfolding of a manifold has been defined and discussed by M. El-Ghoul, in [20]. After that, many papers studied the folding, unfolding and deformation retraction of different types of manifolds; see [19, 21, 23, 24, 25, 28, 74, 77 and 78]. Abu-Saleem, in [76] introduced many results of some geometric transfers of the manifold on the fundamental groups.

In this work, we firstly describe some geometric figures by using the algebra and we study the types of connectedness of geometric figures. Also, we give the effect of some geometric transfers (folding and unfolding) on the homology groups.

During the past fifty years, the Mathematicians have defined different types of algebraic structures and they studied many mathematical applications, such as fuzzy sets, graph theory, n-fold and derivation.

Summary

Imai and Iseki [35, 36] introduced the new two classes of abstract algebras, BCK-algebra and BCI-algebra. BCI-algebra is a generalization of BCK-algebra. Various Mathematicians have studied these algebras extensively and as a result a lot of literature has emerged. Iseki [37] introduced the concept of ideals in BCK-algebras. Chen [16] introduced n-fold positive implicative BCK-algebras. After that, Huang and Chen [33] introduced n-fold implicative BCK-algebras. Many Mathematicians have studied n-fold of some algebraic structures. For example, see [43, 58 and 84].

Prabpayak and Leerawat [69, 70] constructed a new algebraic structure which is called KU-algebra and they introduced the concept of homomorphism of KU-algebra.

In this thesis, we introduce n-fold commutative, n-fold implicative and n-fold positive implicative of KU-algebra as a natural generalization of KU-commutative, KU-implicative and KU-positive implicative of KU-algebra respectively. Also, we study n-fold KU-ideal, n-fold commutative ideal, n-fold implicative ideal and n-fold positive implicative ideal of KU-algebra. Moreover, we obtained a few interesting properties, such as every n-fold KU-positive implicative and n-fold KU-commutative ideal is n-fold KU-implicative ideal. Similarly, the converse is also true.

The concept of fuzzy set was initiated by Zadeh, in [86]. Since then this concept has been applied in many different branches of mathematic such as topological space, vector space, groups, rings and modules.

The study of fuzzy algebraic structures was started with the introduction of the concept of fuzzy groups by Rosenfeld [75]. Xi [83] introduced the notion of fuzzy BCK-algebras. Jun et al [40] studied a fuzzy ideal in BCK-algebra. After that, fuzzy p-ideals and fuzzy H-ideals in BCI-algebras are introduced in [41], [52] respectively. Mostafa et al, in [65] introduced the notion of fuzzy

KU-ideals of KU-algebras and then they investigated several basic properties which are related to fuzzy KU-ideals. Also, n-fold fuzzy sets and its properties are introduced in [44], [46].

In this thesis, we study fuzzy n-fold KU-ideal of a KU-algebra and investigate some related properties. We give a relation between a fuzzy n-fold KU-ideal and a fuzzy ideal. On the other hand, there are several kinds of fuzzy set extensions in the fuzzy set theory, for example, interval-valued fuzzy sets and bipolar-valued fuzzy sets, which were introduced in [45, 55, 56, 64, and 66]. Interval-valued fuzzy sets were first introduced by Zadeh [87] as a generalization of fuzzy sets. Similarly, the concept of bipolar fuzzy sets was initiated by Zhang in [89]. In the final part of this thesis, a generalization of fuzzy n-fold KU-ideals of KU-algebra is studied. Also, the image and the pre-image of interval valued fuzzy n-fold KU-ideal in KU-algebra under homomorphism are introduced. Moreover, the image of bipolar fuzzy n-fold KU-ideal in KU-algebra is given.

The study of graph theory and its properties are topics of interest in algebraic structures. Beck [9] associated to any commutative ring R its zero divisor graphs $G(R)$, whose vertices are the zero divisors of R, with two vertices a, b jointed by an edge in case $ab = 0$. Many Mathematicians studied a graph of a commutative ring by different ways, see [5], [8], [10] and [17]. In [49], Jun and Lee introduced the concept of the associated graph of BCK/BCI-algebra and they proved that if X is a BCK-algebra, then the associated graph of X is connected but if X is a BCI-algebra, then its not connected. Zahiri and Borzooei [88] introduced a new graph of a BCI-algebra X and they defined the concept of a-divisor of BCI-algebra X. But they obtained the same results in [49].

In this thesis, we introduce the notion of graph in KU-algebra and graph folding. We define the KU-graph as the undirected graph with all elements of

KU-algebra X as vertices and for distinct $x, y \in X$, the vertices x and y are adjacent if and only if $x \dot{\wedge} y = 0$. Moreover, we study the graph of equivalence classes of X which is determined by annihilator ideal. Also, by using the definition of graph folding [54], we prove that the graph of equivalence classes and the graph folding of a commutative KU-algebra are the same, and every one, then both, is complete bipartite graph.

This thesis includes five chapters.

In chapter one, the basic definitions and theorems from the topology, graph theory and some algebraic structures are introduced which are useful in the following chapters.

In chapter two, we describe some geometric figures by using the algebra and we study some geometric transfers (folding and unfolding) on this geometric figures. Also, the effect of these geometric transfers on homology groups is discussed. Moreover, the limit of these transfers is studied.

The results of this chapter were published in ″International Journal of Mathematical Archive ″ Vol.4, No.4, 2013, Page 276-281 and ″ Journal of Mathematics Research″ Vol.6, No.4, 2014, Page 90-98.

In chapter three, we introduce some applications of KU-algebra. So, this chapter consists of three sections. In the first one, we introduce the associated graphs of a commutative KU-algebra. Firstly, we define the KU-graph which is determined by all the elements of a commutative KU-algebra as vertices. Secondly, we define the graph of equivalence classes of a commutative KU-algebra by the definition equivalence relation of these vertices, and then the definition of graph folding is applied.

In the second section, we discuss the following concepts of n-fold KU-commutative, n-fold KU-positive implicative, n-fold KU-implicative and n-

fold KU-ideals of KU-algebras as a natural generalization of KU-commutative, KU-positive implicative, KU-implicative and KU-ideals of KU-algebras respectively. Also, we proved that a lot of theorems and properties which related to these concepts.

In the third section, we introduce the concept of a left fixed map in a KU-algebra and we discuss some related properties of this concept. Moreover, we study the notion of left-right (resp., right-left)-derivation in a KU-algebra and establish some results on derivation in a KU-algebra.

The results of this chapter were published in the ″international Journal of Algebra″ Vol.8, No.6, 2014, Page 267-275, ″ Journal of advances in mathematics″ Vol.9, No.7, 2014, Page 2817-2827 and Algebra Letters, No.1 2015.

In chapter four, we define a fuzzy n-fold KU-ideal which is a generalization of the fuzzy KU-ideal of KU-algebra and investigate a lot of theorems and properties which related to this definition. Also, we introduce the concepts of the image and the pre-image of fuzzy n-fold KU-ideal in KU-algebra under homomorphism.

The results of this chapter were published in ″ Annals of Fuzzy Mathematics and Informatics″ Vol.8, No.6, 2014, Page 987-1000.

In chapter five, we introduce a generalization of the fuzzy n-fold KU-ideal of KU-algebras. In the first section, we define the concept of interval value fuzzy n-fold KU-ideal in KU-algebra, which is a generalization of interval value fuzzy KU-ideal of KU-algebra and we obtain few properties which are related to this concept.

In the second section, we study the notion of a bipolar fuzzy n-fold KU-ideal of KU-algebra. Also, we investigate several properties and we give relations between a bipolar fuzzy n-fold KU-ideal and n-fold KU-ideal.

The results of this chapter were published in ″ Journal of mathematical and computational science″ Vol. 5, 2015, No. 2, 246-264 and ″Mathematica Aeterna″ Vol. 4, 2014, No. 6, 633- 650 .

Finally, some algorithms and programs (Qbasic) for KU-algebra, n-fold KU-ideals, fuzzy n-fold KU-ideals in KU-algebra have been constructed.

CHAPTER 1

Basic Concepts