

Lepton Mixing and Flavor Symmetries

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Contents

1	Neutrino masses and finite groups	1
1.1	Fermions in the Standard Model	1
1.2	Dirac and Majorana mass types	3
1.2.1	Dirac mass	4
1.2.2	Majorana mass	6
1.3	Neutrino masses and mixing	11
1.3.1	Models of neutrino masses	11
1.3.2	Neutrino mixing	16
1.4	Experimental results on neutrino masses and mixings	19
1.4.1	Mixing and mass matrices of neutrinos	21
1.4.2	Motivations for horizontal symmetry	23
2	TBM symmetry	25
2.1	Introduction	25
2.2	Deviations of the mass matrix from the TBM form	32
2.2.1	Deviations from the TBM mixing	32
2.2.2	Corrections to the neutrino mass matrix	33
2.2.3	Basis corrections	38
2.2.4	Violation of the TBM conditions	39
2.3	Properties of neutrino mass matrix	49
2.3.1	Normal mass hierarchy	50
2.3.2	Partially degenerate spectrum	55
2.3.3	Inverted mass hierarchy	58

2.3.4	Degenerate spectrum	62
2.4	Deviations from TBM and flavor symmetry	71
2.4.1	Deviations from TBM and new flavor symmetries?	72
2.4.2	Two-component structure of the mass matrix	74
2.4.3	No-symmetry case	76
2.4.4	Matrices with flavor alignment	77
3	$\Delta(27) \ltimes S_2$ discrete symmetry and neutrino masses and mixing	79
3.1	Discrete groups	79
3.1.1	Tensor product of irreducible representation	85
3.2	Top-down and bottom- up	85
3.3	$\Delta(27) \ltimes S_2$ model	88
3.3.1	$\Delta(27)$ multiplication rules	88
3.3.2	The model	92
3.3.3	Vacuum alignment	93
3.3.4	Charged lepton masses	96
3.3.5	Neutrino masses	98
3.4	Quark masses and CKM mixing	106
3.5	Lepton residual symmetries	108
3.6	Connection between residual symmetry and top-down	113
4	The non-tri-bimaximal and $(Z_2)^3$ symmetry	117
4.1	The underlying symmetry of the NTB pattern	122
4.2	The charged-lepton mass matrix	129
4.3	The NTB neutrino mass matrix and type-I seesaw scenario	133
4.3.1	Lepton asymmetry in type-I seesaw scenario	136
4.4	The NTB neutrino mass matrix and type-II seesaw scenario	138
5	Conclusions	143
A	Example of discrete groups	147
	Bibliography	152

List of Figures

2.1	$ \Delta_e $ as a function of s_{13} for different values of $\tilde{\phi}$. We take the best fit values of θ_{23} and θ_{12}	44
2.2	Dependence of $ \Delta_{23} $ on D_{23} for different values of the lightest neutrino mass and $\phi_2 = \frac{\pi}{2}$. We take the best fit values of θ_{13} and θ_{12} . The value $m_3 = 0$ corresponds to the inverted mass hierarchy.	45
3.1	Scatter plot between the angle s_{13} versus the neutrino mass m_3 , all the parameters f_i and u_i vary randomly with each other . . .	105
4.1	Feynman diagrams in SM with right-handed neutrinos that contribute to the decay $N_1 \rightarrow \phi L_\alpha$	137

List of Tables

1.1	The best-fit values, 1σ and 3σ allowed ranges of the neutrino oscillation parameters. NH and IH refer to normal hierarchy and inverted hierarchy respectively [27].	21
2.1	The best fit values and 1σ intervals for the mixing angles according to global oscillation analysis of different groups. The analysis GM-II is based on the high surface metallicity (AGSS09) and modified Gallium cross-section; see [36] for details.	29
2.2	Central values and 1σ allowed intervals for the TBM deviation parameters according to the global analysis of different groups (for more explanation see caption for the Table 2.1).	32
2.3	Special values of the violation parameter Δ_e and the corresponding relations between elements of the mass matrix. Here values of the ratio $s_{13}e^{-i\tilde{\phi}}/\tilde{s}_{13}$ are given for $\alpha = 0$	41
2.4	Special values of the violation parameter $\Delta_{\mu\tau}$ and the corresponding relations between the elements of the mass matrix. Values of the ratio D_{23}/\tilde{D}_{23} are given for $\beta = 0$	46

2.5	Numerical examples of neutrino mass matrices in the cases of normal mass hierarchy (NH), partially degenerate spectrum (PD), inverted hierarchy (IH) and degenerate spectrum (D). The numbers in brackets of the scenario definition indicate the CP-phases (ϕ_2, ϕ_3) . We show matrices for the exact TBM (left column), the best fit values of mixing angles (central column) and mixing angles allowed at 1σ level (right column). We take $\delta = 0$ and the elements of the matrices are in the unit 10^{-2} eV.	51
2.6	The same as in Table 2.5 for the Dirac phase $\delta = \pi$.	52
3.1	The character table of $\Delta(27)$.	91
3.2	The singlet multiplications of the group $\Delta(27)$.	91
3.3	Field transformations under $\Delta(27)$, and Z_4 . Here α refers to 1, 2, 3.	92
3.4	Quark assignments under $\Delta(27)$ and Z_4 .	107
4.1	Present [34], [28] global-fit results for the three neutrino mixing angles $(\theta_{12}, \theta_{23}, \theta_{13})$, with their 3σ ranges (95% C.L.). We show also the corresponding values in the generic and the special NTB patterns, and in the TBM pattern.	121
A.1	Characters of A_4 representations	148

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Summary

The masses in the quark sectors are set in strong hierarchy while the hierarchy is smaller in the lepton sector. The mixing in the lepton sector is large (one mixing angle is close to maximal, one is large and the third is small), while the mixing in the quark sector is small. The horizontal or flavor symmetry was proposed to give answers to all above mentioned observations. In this thesis we study the lepton mixing and how to infer the flavor symmetry from it. First we study the widely proposed mixing which is called tri-bimaximal (TBM) and try to answer the question "Is TBM an accidental symmetry". The TBM mixing is not accidental if structures of the corresponding leptonic mass matrices follow immediately from a certain (residual or broken) flavor symmetry. We study the effect of the deviation of the mixing parameters (angles and phase) from their TBM values on the TBM symmetry which is the symmetry manifesting in the mass matrix. We show that possible deviations from the TBM mixing can lead to strong modifications of the mass matrix and strong violation of the TBM mass relations. As a result, the mass matrix may have an "anarchical" structure with random values of elements or it may have some symmetry which differs from the TBM symmetry. Interesting examples include matrices with texture zeros, matrices with certain "flavor alignment" as well as hierarchical matrices with a two-component structure, where the dominant and sub-dominant contributions have different symmetries. This opens up new approaches to understand lepton mixing.

We study two methods to deal with flavor symmetry. The first is the conventional way called (top - down) in which one can consider the invariance of the Lagrangian under a certain group and finally try to obtain the desired mixing and masses to check whether this group can be considered as the flavor symmetry group or not. Following this method we study a model based on the discrete group $\Delta(27) \times S_2$ account for the recent neutrino oscillation data that deviate from TBM. Breaking of the $\Delta(27)$ group with a certain vacuum

alignment can lead to the observed deviations from TBM mixing, in particular nonzero 1-3 mixing and deviation of the 2-3 mixing from maximal. Some correlations between mixing angle deviations from TBM resulted from the model consistent with the recent neutrino data.

The other method is called (bottom - up) in which we study the residual symmetry that manifests itself in the mass matrix and how to use it to obtain the flavor symmetry group. The connection between the two methods is studied also.

We then study another model based on the group $(Z_2)^3$ for the non-vanishing value for the smallest mixing angle (θ_{13}), we derive and find explicit realizations of the $(Z_2)^3$ flavor symmetry which characterizes, for the neutrino mass matrix, uniquely a variant of the tripartite form, originally conceived to lead to TBM mixing with $\theta_{13} = 0$, so as to allow now for a non-tri-bimaximal pattern with non-zero θ_{13} . We impose this flavor symmetry in a setting including the charged leptons and we see that it can make room, through higher order terms involving new SM-singlet scalars, for the mass hierarchy of charged leptons. Moreover, within a type-I seesaw mechanism augmented with the flavor symmetry, certain patterns occurring in both the Dirac and the Majorana neutrino mass matrices can accommodate all types of mass hierarchies in the effective neutrino mass matrix, but no lepton/baryon asymmetry can be generated. Finally, we discuss how a type-II seesaw mechanism, when supplemented with the flavor symmetry, could be used to interpret the observed baryon asymmetry through leptogenesis.

Introduction

The flavor symmetry is one of the important issues in particle physics. The elementary particles are classified into leptons and quarks, and can be sorted in three families. The corresponding particles in the three families are identical in all properties except mass. There are many puzzles in particle physics such as why there are three families of elementary particles? Why is the mixing in the lepton sector large while it is small among quarks? Why the mass spectrum is strongly hierarchical in quarks, while it is weakly hierarchical for neutrinos? What are the interpretations of some relations between the mass ratios and mixing angles? The previous questions motivate particle physicists to propose that elementary particles are subject to horizontal or flavor symmetry beside the conventional gauge symmetries. The horizontal symmetry means the symmetry between the particle and its corresponding particles in the other families, for example the symmetry between e , μ and τ . Authors investigated many groups to check whether it can account for the experimental data on neutrino masses and mixing. The discrete groups were widely used to avoid appearance of Goldstone bosons when symmetry is broken.

The so called Tri-bimaximal (TBM) mixing was proposed in (2002) [1] to be the lepton mixing matrix. The past neutrino oscillation data (see for instance [2]) indicated that the observed mixing angles were close to the TBM values. The recent data shows big deviations from TBM values. We investigate whether the TBM is accidental or not by studying the influence of the deviations from TBM values on the symmetry manifesting itself in the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal. If the symmetry of the mass matrix is broken completely, one can say that the TBM symmetry is accidental and it results from an interplay of different and to a large extent independent factors or/and contributions.

There are two scenarios to deal with flavor symmetries in particle physics.

The popular one is called (top - down) based on considering the invariance of the Lagrangian under a certain group G . After spontaneous symmetry breaking via scalar fields, the mass matrix will be constructed and the mixing parameters can be calculated. Comparing the the calculated masses and mixing parameters with the experimental neutrino oscillation data, one can examine whether the group G is the correct flavor group or not.

The second scenario called (bottom - up) depends on the residual symmetry manifesting in the mass matrix. It is assumed that the residual symmetry is a consequence of a flavor symmetry group G which depends on the mixing matrix U . The flavor group G can be constructed from the residual symmetry of the mass matrix by considering the elements of the residual symmetry as the basis of G .

For the specific models, we consider two examples. The first one is based on the group $\Delta(27) \times S_2$ and we use it to give an account of the recent neutrino oscillation data that deviates from TBM. The second model is based on the group $(Z_2)^3$ ensuring a non-vanishing value for the smallest mixing angle (θ_{13}). We use this model to discuss how a type-II seesaw mechanism, when supplemented with the flavor symmetry, could be used to explain the observed baryon asymmetry through leptogenesis.

The thesis is organized as follows.

In chapter 1 we give the basic structure of the SM, describe its particle content and how to construct fermion masses. We show that it is possible for neutral particles like neutrinos to construct two types of mass terms called Dirac type and Majorana type. We also discuss the different methods to construct neutrino mass like the effective non renormalizable 5-dimensional term, type-I seesaw mechanism and type-II seesaw mechanism. The neutrino mixing is also studied and the popular mixing matrices like tri-bimaximal and bi-maximal matrices are considered. We also show the recent neutrino oscillation data on mass square difference and mixing.

In chapter 2 we present a simple formalism which accounts for the effects of deviations from the TBM on the structure of the neutrino mass matrix.