Lepton Mixing and Flavor Symmetries

A Thesis submitted
to

Faculty of Science, Ain Shams University
In a partial fulfilment for the requirements of the PhD Degree in Science (Physics)
Presented By

Mohammed Ahmed Abbas Abd AlGalil
Physics Department, Faculty of Science, Ain Shams University

Supervision Committee

1- **Prof. Shaaban Said Khalil**
Mathematics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.

2- **Prof. Alexei Yu. Smirnov**
Max-Planck Institute for Nuclear Physics, Heidelberg, Germany
The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34014 Trieste, Italy

3- **Dr. Elsayed Ibrahim Lashin**
Physics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.

Physics Department, Faculty of Science,
Ain Shams University
Cairo, Egypt
2015

Approval Sheet

Title of the PhD. Thesis

Lepton Mixing and Flavour Symmetries

Name of Candidate
Mohammed Ahmed Abbas Abd AlGalil,
Physics Department, Faculty of Science, Ain Shams University, Cairo, Egypt

Submitted to the
Faculty of Science, Ain Shams University, Cairo, Egypt

Supervision Committee

1- Prof. Shaaban Said Khalil
Mathematics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.
Centre for Theoretical Physics, Zewail City of Science and Technology, Cairo, Egypt.

2- Prof. Alexei Yu. Smirnov
Max-Planck Institute for Nuclear Physics, Heidelberg, Germany
The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, I-34014 Trieste, Italy

3- Dr. Elsayed Ibrahim Lashin
Physics Department, Faculty of Science, Ain Shams University, Cairo, Egypt.
Centre for Theoretical Physics, Zewail City of Science and Technology, Cairo, Egypt.

Physics Department, Faculty of Science,
Ain Shams University
Cairo, Egypt
2015

Contents

List of Figures

List of Tables

Acknowledgements

I am very thankful to **Allah**, by the grace of whom the progress of this work was possible. My gratitude, thanks and appreciation to my parents who have the first favor, after **Allah**. They offered me the advice, hope and support to establish this work.
I would like to thank Prof.Mostafa Shalaby for his time, scientific guidance, his advice and support. My sincere gratefulness to him for his encouragement during the course of this thesis.
I would like to express my sincere gratefulness to my supervisor Prof. Alexi Smirnov for building the important background in the neutrino physics, suggesting the problem, stimulating discussion and valuable encouragement during the course of this work.
Also, I want to send my deep thanks to Prof. Shaaban Khalil for his support, encouragement and building the most essential background in our field of research, the Quantum Field Theory.
I express my deep thanks to Dr. Elsayed Lashin for his stimulating discussion, valuable encouragement, his help and support. He spent much time to help me to finish my thesis.

I would like to thank the International Center for Theoretical Physics (ICTP), Trieste, Italy, and Sandwich Training Educational Program (STEP) for hosting me and providing all the necessary needs wanted to complete my work.
I would like to thank the center for fundamental physics (CTP), Zewail city,

Egypt where part of this thesis was accomplished. I wish also to express my gratefulness to Physics Department , Faculty of Science, Ain Shams University, for providing me with the Ph.D. courses that helped me a lot.
Also I want to thank all who taught me in the undergraduate stage in Ain Shams University for building a good background in the theoretical physics.

Summary

The masses in the quark sectors are set in strong hierarchy while the hierarchy is smaller in the lepton sector. The mixing in the lepton sector is large (one mixing angle is close to maximal, one is large and the third is small), while the mixing in the quark sector is small. The horizontal or flavor symmetry was proposed to give answers to all above mentioned observations. In this thesis we study the lepton mixing and how to infer the flavor symmetry from it. First we study the widely proposed mixing which is called tri-bimaximal (TBM) and try to answer the question "Is TBM an accidental symmetry". The TBM mixing is not accidental if structures of the corresponding leptonic mass matrices follow immediately from a certain (residual or broken) flavor symmetry. We study the effect of the deviation of the mixing parameters (angles and phase) from their TBM values on the TBM symmetry which is the symmetry manifesting in the mass matrix. We show that possible deviations from the TBM mixing can lead to strong modifications of the mass matrix and strong violation of the TBM mass relations. As a result, the mass matrix may have an "anarchical" structure with random values of elements or it may have some symmetry which differs from the TBM symmetry. Interesting examples include matrices with texture zeros, matrices with certain "flavor alignment" as well as hierarchical matrices with a two-component structure, where the dominant and sub-dominant contributions have different symmetries. This opens up new approaches to understand lepton mixing.

We study two methods to deal with flavor symmetry. The first is the conventional way called (top - down) in which one can consider the invariance of the Lagrangian under a certain group and finally try to obtain the desired mixing and masses to check whether this group can be considered as the flavor symmetry group or not. Following this method we study a model based on the discrete group $\Delta(27) \ltimes S_2$ account for the recent neutrino oscillation data that deviate from TBM. Breaking of the $\Delta(27)$ group with a certain vacuum

alignment can lead to the observed deviations from TBM mixing, in particular nonzero 1-3 mixing and deviation of the 2-3 mixing from maximal. Some correlations between mixing angle deviations from TBM resulted from the model consistent with the recent neutrino data.

The other method is called (bottom - up) in which we study the residual symmetry that manifests itself in the mass matrix and how to use it to obtain the flavor symmetry group. The connection between the two methods is studied also.

We then study another model based on the group $(Z_2)^3$ for the non-vanishing value for the smallest mixing angle (θ_{13}), we derive and find explicit realizations of the $(Z_2)^3$ flavor symmetry which characterizes, for the neutrino mass matrix, uniquely a variant of the tripartite form, originally conceived to lead to TBM mixing with $\theta_{13} = 0$, so as to allow now for a non-tri-bimaximal pattern with non-zero θ_{13}. We impose this flavor symmetry in a setting including the charged leptons and we see that it can make room, through higher order terms involving new SM-singlet scalars, for the mass hierarchy of charged leptons. Moreover, within a type-I seesaw mechanism augmented with the flavor symmetry, certain patterns occurring in both the Dirac and the Majorana neutrino mass matrices can accommodate all types of mass hierarchies in the effective neutrino mass matrix, but no lepton/baryon asymmetry can be generated. Finally, we discuss how a type-II seesaw mechanism, when supplemented with the flavor symmetry, could be used to interpret the observed baryon asymmetry through leptogenesis.

Introduction

The flavor symmetry is one of the important issues in particle physics. The elementary particles are classified into leptons and quarks, and can be sorted in three families. The corresponding particles in the three families are identical in all properties except mass. There are many puzzles in particle physics such as why there are three families of elementary particles? Why is the mixing in the lepton sector large while it is small among quarks? Why the mass spectrum is strongly hierarchial in quarks, while it is weakly hierarchical for neutrinos? What are the interpretations of some relations between the mass ratios and mixing angles? The pervious questions motivate particle physicists to propose that elementary particles are subject to horizontal or flavor symmetry beside the conventional gauge symmetries. The horizontal symmetry means the symmetry between the particle and its corresponding particles in the other families, for example the symmetry between e, μ and τ. Authors investigated many groups to check whether it can account for the experimental data on neutrino masses and mixing. The discrete groups were widely used to avoid appearance of Goldstone bosons when symmetry is broken.

The so called Tri-bimaximal (TBM) mixing was proposed in (2002) [1] to be the lepton mixing matrix. The past neutrino oscillation data (see for instance [2]) indicated that the observed mixing angles were close to the TBM values. The recent data shows big deviations from TBM values. We investigate whether the TBM is accidental or not by studying the influence of the deviations from TBM values on the symmetry manifesting itself in the neutrino mass matrix in the basis where the charged lepton mass matrix is diagonal. If the symmetry of the mass matrix is broken completely, one can say that the TBM symmetry is accidental and it results from an interplay of different and to a large extent independent factors or/and contributions.

There are two scenarios to deal with flavor symmetries in particle physics.

The popular one is called (top - down) based on considering the invariance of the Lagrangian under a certain group G. After spontaneous symmetry breaking via scalar fields, the mass matrix will be constructed and the mixing parameters can be calculated. Comparing the the calculated masses and mixing parameters with the experimental neutrino oscillation data, one can examine whether the group G is the correct flavor group or not.

The second scenario called (bottom - up) depends on the residual symmetry manifesting in the mass matrix. It is assumed that the residual symmetry is a consequence of a flavor symmetry group G which depends on the mixing matrix U. The flavor group G can be constructed from the residual symmetry of the mass matrix by considering the elements of the residual symmetry as the basis of G.

For the specific models, we consider two examples. The first one is based on the group $\Delta(27) \ltimes S_2$ and we use it to give an account of the recent neutrino oscillation data that deviates from TBM. The second model is based on the group $(Z_2)^3$ ensuring a non-vanishing value for the smallest mixing angle (θ_{13}). We use this model to discuss how a type-II seesaw mechanism, when supplemented with the flavor symmetry, could be used to explain the observed baryon asymmetry through leptogenesis.

The thesis is organized as follows.

In chapter 1 we give the basic structure of the SM, describe its particle content and how to construct fermion masses. We show that it is possible for neutral particles like neutrinos to construct two types of mass terms called Dirac type and Majorana type. We also discuss the different methods to construct neutrino mass like the effective non renormalizable 5-dimensional term, type-I seesaw mechanism and type-II seesaw mechanism. The neutrino mixing is also studied and the popular mixing matrices like tri-bimaximal and bi-maximal matrices are considered. We also show the recent neutrino oscillation data on mass square difference and mixing.

In chapter 2 we present a simple formalism which accounts for the effects of deviations from the TBM on the structure of the neutrino mass matrix.