Ain Shams University
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On Using Computational Geometry Algorithms in Computer Graphics

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Summary

The main object of this thesis is to study the application of computational geometry algorithms in computer graphics. We have chosen the set of fractal objects as a representative for computer graphics. This thesis consists of an introduction, five chapters, and an appendix. It is organized as follows:

In the first chapter: we introduce the basic concepts of computational geometry and its applications. Then we discuss some of its important problems such as *Convex hull*, *Point Location*, *Closest Pair*, *Voronoi Diagram*, *and Polygon Triangulation*. We discuss in detail the problem of convex hull in the plane. Then we present some algorithms for solving this problem in 2-dimensional space such as *Graham scan* and *QuickHull techniques*. We also present the *Gift-wrapping* and *QuickHull algorithms* for solving this problem in three or more dimensional space. Finally we study the triangulation problem for a set of points and present some algorithms for solving this problem such as Bowyer-Watson and Projection algorithms.

In the second chapter: we introduce a brief introduction to the fractal geometry. Then we discuss the basic types of fractal geometry which are iterated function systems (IFS) fractals and complex fractals. We also present some of fractal objects such as

Sierpinski triangle, *Sierpinski carpet*, *Cantor set*, *Koch curve*, and *Fern leaf*. Then we describe how we can create these fractals using IFS in 2-dimensional space.

In the third chapter: we introduce the basic definition of the Lindenmayer system (L-system) which is considered one of the most common techniques used for generating fractal objects. Then we discuss the different types of L-systems such as *Deterministic Context-free L-system*, *Stochastic L-system*, *Parametric L-system*, *and Context-sensitive L-system.* We also describe how we can use L-systems technique to model plants such as trees in 2-dimensional and 3-dimensional space.

In the fourth chapter: we introduce our main new results. In this chapter, we present a new algorithm that generates a 3D fractal plant according to the given 3D L-system and also creates the triangulation shape of the vertices generated from the last stage of the L-system rewriting process. The resulting shapes can be used as surfaces models for architectural objects. The results of this chapter are original and are published in The International Conference on Computational Geometry (ICCG), Venice, Italy, (29-31/ 10/2008).

In the fifth chapter: we give the conclusion about this work and we also introduce some plans for the future work.

The appendix includes three sections. The first section contains the source codes that are used to generate some of the fractal objects discussed in Chapter two using IFS. The programs of the first section are written in the Java applet language. The second section contains the source codes that are used to generate the fractal objects discussed in Chapter three using L-systems. The programs of the second section are written in the Matlab language. The third section contains an implementation of the iterative steps of the algorithm presented in Chapter four and the program written in the Matlab language.

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INTRODUCTION

Geometric objects such as points, lines, and polygons are the basis of a broad variety of important applications and give rise to an interesting set of problems and algorithms. The name geometry reminds us of its earliest use: for the measurement of land and materials. Today, computers are being used more and more to solve larger-scale geometric problems. Over the past two decades, a set of tools and techniques has been developed that takes advantage of the structure provided by geometry. This discipline is known as computational geometry.

Computational geometry is the branch of computer science that studies algorithms in order to solve geometric problems. It was named and largely started around 1975 by Shamos, whose Ph.D. thesis[44] attracted considerable attention. After a decade of development, the field came into its own in 1985, when three components of any healthy discipline were realized: a textbook, a conference, and a journal. In 1985, Preparata and Shamos introduced their book "*Computational Geometry: An Introduction*[32]" which is considered the first textbook solely devoted to the topic. Since 1985, several texts, collections, and monographs have appeared([27], [43], [12], [29], [10], [28]).

In modern engineering and mathematics, computational geometry has applications in, among other fields, computer

graphics, robotics, computer-aided design, and statistics. The input to a computational-geometry problem is typically a description of a set of geometric objects, such as a set of points, a set of line segments, or the vertices of a polygon in counterclockwise order. The output is often a response to a query about the objects, such as whether any of the lines intersect, or perhaps a new geometric object, such as the convex hull (smallest enclosing convex polygon) of the set of points.

One of the new branches which can be studied using the algorithms of computational geometry is the fractal geometry.

Fractal geometry is a new branch of mathematics used to model natural objects which cannot be easily represented by the Euclidean geometry. Although Cantor, Van Koch and Sierpinski discovered the first fractal objects at the beginning of the 20th century, the real development of the theory of fractals was started in the 1970s by a French mathematician of Polish origin – Benoit Mandelbrot. In 1982, Mandelbrot published his book "*The Fractal Geometry of Nature*[24]" that demonstrated the potential application of fractals to nature and mathematics. Through his computer experiments Mandelbrot also developed the idea of reconstructing natural scenes on computer screens using fractals. Also, the use of a computer enabled him to create, among the others, a graphical image of the most famous object of contemporary mathematics, which came to be called Mandelbrot's set, after its creator.