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بالرسالة صفحات
لم ترد بالأصل

ON BI-LEVEL LINEAR FRACTIONAL PROGRAMMING PROBLEMS

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2006

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I certify that this work has not been accepted in substance for any academic degree and is not being concurrently submitted in candidature for any other degree.

Any portions of this thesis for which I am indebted to other sources are mentioned and explicit references are given.

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Approval Sheet

On Bi-Level Linear Fractional Programming Problems

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**A Thesis submitted as Partial Fulfillment for the Degree
for Master of Science in Operations Research.**

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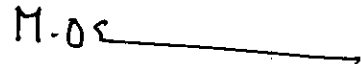
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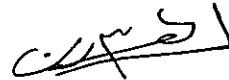
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IV.

To my parents, my wife who gave me her full time and the support during my study and to my brother Sobhy.

ACKNOWLEDGEMENT

I am deeply thankful to **ALLAH** by grace of whom, the progress and success of this thesis was possible.

I am grateful to **Prof. Dr. Mohamed Sayed Ali Osman**, professor of Mathematics and Operations Research, Vice-dean of the Higher Technological Institute, Tenth of Ramadan City, for his boundless encouragement and instructive assistance throughout this thesis.

I would like to thank **Prof. Dr. Shaban Abdul-Hamed Shaban**, Professor of Mathematical Statistics; Institute of Statistical Studies and Research, Cairo University, for his great advice and help.

I would like to express my deep thanks for **Dr. Aly Mohamed Ragab**, Assistant Prof., Department of Operations Research, Institute of Statistical Studies and Research, Cairo University, for his co-operation, continuous interest and patience.

Also, I would like to thank **Prof. Dr. Mahmoud Attia Abo-Sinna**, professor of Mathematics and Operations Research, Faculty of Engineering, Menofia University, for his help to choose the research point and his great support.

Finally, would like to thank **Dr. Osama Ezz-Eldin Emam**, Lecturer of mathematics, the Higher Technological Institute, Tenth of Ramadan City, for his great advice and help.

ABSTRACT

The decentralized planning has been recognized as an important decision-making problem. Multi-level programming models partition control over decision variables among the ordered levels within a hierarchical planning structure. The decentralized planning seeks to find a simultaneous compromise among the various objective functions of the different divisions.

The Bi-level linear programming problem is a special case of the multi-level linear programming problems and is a nested optimization model involving two problems, an upper and lower one. Both problems have to be optimized given a single feasible region. Each decision-maker at both levels attempts to optimize his individual objective function, and their final decisions are executed sequentially where the upper-level decision-maker makes his decision firstly.

Most applications of the bi-level linear programming problems are in the economics realm, particularly central economic planning. The problem can be viewed as a two-person sequential game of perfect information, i.e., a two-person static Stackelberg game, where two players wish to minimize their own cost functions. The first player, the leader, knows the cost function of the second player, the follower, who may or may not know the cost function of the leader, knows the selected strategy by the leader and takes this into account when computing his own strategy. The leader is assumed to be able to anticipate the reactions of the follower.

When there is only one level of decision, the optimization problems involving one or more ratios in the objective function are called fractional programming. Ratio functions arise in economic applications when an efficient measure of a system is optimized or in approaching a stochastic programming problem.

In the bi-level linear programming, the linear fractional objectives are sometimes encountered (i.e., ratio objectives that have linear numerators and denominators). Examples of fractional objectives including return on investment, liquidity, productivity, assets per share, etc; can be found in finance or corporate planning.

Chapter one attempts to present a survey on the bi-level linear programming problems, its definition, formulation, properties, geometric characterization and solution approaches. For solving the bi-level linear programming problem, the *Kuhn-Tucker* approach, the Parametric Complementary Pivot Algorithm and the "Kth-Best Algorithm" are presented. The linear fractional programming is presented in this chapter to show its formulation, assumptions and solving methods. Some examples of objective functions or criteria which can be represented as bi-level linear fractional programming problems are showed also.

Chapter two will propose a new algorithm for the stability set of the first kind for the bi-level linear fractional programming problems under using an interactive fuzzy programming approach for the bi-level linear fractional programming problems with the essentially co-operative decision-makers. In this interactive fuzzy programming approach, after determining the fuzzy goals of the decision-makers at both levels, a satisfactory solution is efficiently derived by updating the satisfactory level of the upper-level decision-maker with considerations of overall satisfactory balance between both levels. In the interactive process, the solution is obtained by combined use of the bisection method and a linear programming technique and using the variable transformation method for dealing with the fractional objective functions.

Chapter three presents the concepts of the fuzzy programming with the fuzzy set theory. By using the tolerance concept of the membership function, a new fuzzy approach for solving the bi-level linear fractional programming problems is proposed, where the upper-level decision-maker defines his objective function and decision variables with possible tolerances which are described as membership functions by the fuzzy set theory. We will propose a new algorithm for the stability set of the first kind for the bi-level linear fractional programming problems under using the proposed fuzzy approach. Finally, chapter four presents conclusions and some points for future researches.

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