بسم الله الرحمن الرحيم

شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

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ALEXANDER-SPANIER COHOMOLOGY

OF K-TYPES

A Thesis
Submitted for the degree
of
(Ph.D.) in Science
(Pure Mathematics)

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Cairo, Egypt
1998

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ACKNOWLEDGMENT

Praise and gratitude be to the almighty **God** who have gifted me with inspiration to write and complete this thesis.

I would like to give a special thanks to **Prof. Dr. Abd El-Sattar A. Dabbour,** Deparment of Mathematics, Faculty of Science, Ain Shams University, for his continuous supervision, encouragement, assistance throughout my enitre work, and for giving so generously of his time to complete this thesis in the final form. Without his guidance and moral support this thesis would have been extremely difficult to write.

Again many thanks to **Prof. Dr. Entessarat M. El-Shobaky**, Head of Department of Mathematics , Faculty of science, Ain Shams University , for great help, kind, and understanding, during my stay in Eygpt.

Also, many thanks and gratitude to **Dr. Hamza A. Abojabal**, Head of Department Mathematics , Faculty of Science, King Abdulaziz University (Jeddah), who was a big support , backing and directing my procedures.

Finally, I'm very glad to mention the **Computer Department**, Faculty of Science, Ain Shams University, and thank them for their great hospitality and all the facilities I've recieved there.

Above all, the greatest thanks for my dear husband, Mr. Jamal Al. Nuwaiser, who gave me the whole enthusiasm to fulfil my duty. I really do appretiate your great love and big support.

SUMMARY

It is well-known that it has been given many definitions for extending the cohomology theory of complexes to arbitrary topological spaces; the two theories most commonly used are the $\check{C}ech$ and singular theories, [19],[27], [37]. A different definition was given by J.W.Alexander, [9], who constructed cohomology group for a compact metric space using functions of sets of point in the space. With a suitable choice of coefficient groups, the cohomology groups obtained by Alexander were the character groups of the Vietories homology groups, [12], [39]. Since for a compact metric space the $\check{C}ech$ groups and the Vietories groups are identical, Alexander's construction gives the $\check{C}ech$ cohomology groups for such a space. As the definition involving functions is simpler than the definitions in terms of covers usually used to obtain the $\check{C}ech$ groups, a generalization of Alexander's definition applicable to arbitrary space is desirable. A.D.Wallace ,[40], has suggested such definition which is valid for any space and is even some what simpler than Alexander's definitions.

E.Spanier in his paper, [36], has concerned with the question of how the groups defined by Wallace compare with the cohomology groups obtained by different constructions. The new groups agree with the $\check{C}ech$ cohomology groups on the category of compact Hausdorff spaces. This implies that they differ from the singular cohomlogy groups as it is known that there exists compact Hausdorff space on which the $\check{C}ech$ and singular theories differ. For compact spaces the Spanier's groups don't agree with the $\check{C}ech$ groups. Since the singular groups of infinite locally finite complexes agree with those based on infinite cochains, [36], the new groups coinicide with the singular ones for such spaces. Thus the Spanier's definition gives rise to an extension of cohomology from polyhedra to general spaces which is distinct from the known extensions.

At the present time the resulting cohomology theory is called Alexander -Spanier cohomology theory, denoted by $\bar{H}^*$. For deeper properties of the Alexander-Spanier theory, E.Spanier has introduced the cohomology of a space with coefficients in a presheaf, [37]. By using the general properties of this cohomology it is proved that on the category of paracompact spaces the Alexander-Spannier and $\check{C}ech$ cohomologies are isomorphic. This cohomology of presheaves has been also applied to compare the singular and Alexander-Spanier cohomology theories; it is

proved that they are isomorphic for manifolds.

As it is known that if the group of coefficient in the theory of the Alexander-Spanier cohomology or in the isomorphic theory of the *Čech* cohomology is a topological group, then the definitions of these cohomologies either don't take into account the toplogy of it, [19], [37], or consider only the case of compactness for the coefficient group and by means of this group a topology is introduced in the corresponding cochain complexes and the resulting cohomologies will be compact groups, [13], [16]. Starting from this point of view, continuous cohomology naturally arises when the coefficient group of a cohomology theory is topological , [14], [15], [23]. Recently, Mdzinarishvili, [31], introduced a new cohomology of Alexander-Spanier type with coefficients in an arbitrary topological abelian group. It coincides with the Alexander-Spanier cohomology when the coefficient group has indiscrete (trivial) topology. Mdzinarishvili cohomology can be considered as a variant of the continuous cohomology of a space with two topologies in the sense of Bott-Hoefliger, [32], also it is isomorphic to the continuous cohomology of a simplicial space defined by Brown-Szczarba, [14]. By means of the idea of K-groups, [2], [11], a number of homology and cohomology theories are generalized and discussed on different categories of topological spaces over pairs of coefficient groups, [3], [4], [5]. A main advantage of using a pair (G, G') of coefficient abelian groups is the possibility of considering the finite cohomology.

In the present work , we introduce K-types of the Alexander-Spanier cohomology with coefficient in a pair of abelian groups . So we obtain a generalization of the Alexander-Sapnier cohomology theory and its partially continuous type. Also, these generalizations are expressed as limit of cochain direct systems. Moreover, we are led to study these K-types from the point of view of Eilenberg-Steenrod axioms, [19]. The verification of these all axioms, save for Homotopy and Excision, are easy consequences of the corresponding theorms about cochain complexes.The dimension axiom is considered from another point of view. To prove Homotopy axiom, we use the idea that our cohomology groups are expressed as direct limits of cohomologies on the directed set of open coverings, and a cochain homotopy is constructed from the given homotopy . The Excision axiom for the partially continuous K-type is given for the compact Hausdorff spaces and over a pair of absolute retract coefficient groups. Therefore the uniqueness theorm of the cohomology theory on the category of compact polyhedral pairs, [19], asserts that Alexander-Spanier K-types with coefficients in a pair of absolute retract topological abelian groups are naturally isomorphic. The present work is concluded by discussing some interesting properties for Alexander-Spanier cohomology of $K-$types, namely, Tautness and Continuity properties. Lastly, some applications of these K-types are given.

The thesis consists of four chapters :

Chapter I, presents the majority of the prime knowledge that we will use refer to the rest of the other chapters. The concept of a cohomology theory is stressed here, direct and inverse systems of groups and their limit groups. The last part of chapter I is devoted to give both Alexander-Spanier cohomology theory $\bar{H}^*$ and Mdzinarishvili cohomology construction $\bar{h}^*$, briefly.

In the first section of chapter II, by using the idea of $K-$groups it is given the construction of $K-$ Alexander-Spanier cohomology $\bar{H}_K^*$ over a pair (G, G') of (discrete) coefficient groups which is a generalization of the Alexander-Spanier cohomology theory.
In the second section, by taking in consideration that the pair (G, G') of coefficients admits an arbitrary topological abelian groups, and by using the idea of mapping cone we pass to another cohomology construction which is called partially continuous K- Alexander-Spanier cohomology denoted by $\tilde{H}_K^*$. On certain conditions our construction $\tilde{H}_K^*$ is naturally isomorphic with $\bar{h}^*$.
This chapter is concluded by showing that if we admite X to be a discrete topology then $\tilde{H}_K^q(X; G, G') = \bar{H}_K^q(X; G, G')$.
The main results of chapter II are included in [6], [7].

Chapter III, is devoted to study the two constructions $\bar{H}_K^*$, $\tilde{H}_K^*$ from the point of view of seven axioms of Eilenberg-Steenrod for a cohomology theory. The first four axioms are verified in the first section of this chapter. In the second section, using the idea of $\bar{\alpha}-\bar{\beta}-$ contiguous maps, which induces a cochain homotopic maps, the homotopy axiom (axiom 5) is proved for compact spaces. The last two axioms (Excision and Dimension axioms) are discussed in the third section. For the second construction $\tilde{H}_K^*$ the Excision axiom holds, provided (X, A) is a pair

of compact Hausdarff spaces, and (G,G') consists of absolute retract topological abelian groups.
It is proved that $\bar{H}^*_K$, $\check{H}^*_K$ are isomorphic on the category of compact polyhedral pairs when (G,G') is a pair of absolute retract topological abelian groups.
The main results of chapter III are included in [8].

The first and the second section of chapter IV are centered arround the Tautness and Continuity properties for both Alexander-Spanier cohomology of K-Types. In the third section some applications of the Alexander-Spanier of K-types are given. The first application is studying the $K-$Alexander-Spanier cohomology at zero dimension, and its relation with the topological property of connectedness. The second application is giving the concept of the partially continuous K-Alexander-Spanier cohomology of an Excision map and then calculate its value at dimension 0, 1. The third application is showing how the pair (G,G') plays an essential role in calculating some cohomology groups.
Some results of this chapter are included in [7].