
شبكة المعلومات الجامعية


# بسم الله الرحمن الرحيم


# شبكة المعلومات الجامعية

# التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

# جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

## قسم

نقسم بالله العظيم أن المادة التي تم توثيقها وتسجيلها
علي هذه الأفلام قد أعدت دون أية تغيرات

## يجب أن

تحفظ هذه الأفلام بعيدا عن الغبار

في درجة حرارة من ١٥-٢٥ مئوية ورطوبة نسبية من ٢٠-٤٠%

**To be Kept away from Dust in Dry Cool place of 15-25- c and relative humidity 20-40%**

# بعض الوثائـــــــق الاصليـــــة تالفـــة

# بالرسالة صفحات لم ترد بالاصل

# *OPERATIONS ON BITOPOLOGIES*

# *VIA IDEALS*

*A THESIS*
*SUBMITTED TO FACULTY OF SCIENCE*
*TANTA UNIVERSITY*

*For The Degree of Doctor of Philosophy in Science*

*(Pure Mathematics)*

*By*

*Amany Mohammad Mohammad Abd-El Aziz*

*Assistent Lecturer of pure Mathematics,*
*Department of Mathematics ,*
*Al- Arish Faculty of Education*
*Suez- Canal University*

*1998*

# *SUPERVISORS*

***Prof.Dr. MOHAMMAD EZZAT ABD EL-MONSEF***

***Professor of Pure Mathematics ( Topology )***

***Vice Dean , Faculty of Science ,***

***Tanta University .***

***Prof. Dr. SNAA MOHAMMAD EL-ASSAR***

***Assistant Professor of Pure Mathematics***

***Faculty of Science ,***

***Tanta University .***

# *CURRICULUM VITAE*

***Name*** : Amany Mohammad Mohammad Abd - El Aziz

***Date of Birth*** : 25 - 7 - 1957

***Locality*** : Cairo

***Nationality*** : Egyptian

***Qualifications*** : B.Sc. degree in Mathematics ,in 1979 , Faculty of Education, Ain shams University ,
B.Sc. degree in Mathematics , 1981 , Faculty of Science, Ain shams University ,
M.Sc. degree in ( Pure Mathematics ) 1986 ,
Faculty of Science , Ain shams University .

***Experiences*** : Demonstrator and Assistant Lecturer at Department of Mathematics , Insurance and Statistics Sadat Academy for management Sciences.
Assistant Lecturer at Department of Mathamatics , Faculty of Education , El -Arish , Suez Canal University .

***Head of Department of Mathematics***
***Prof. Dr.***
M. O Shakery

*This Thesis is dedicated*

*to the great topologist ( late )*

*Prof . Dr. A.S. Mashhour*

# *ACKNOWLEDGEMENTS*

First and formost all gratitude and thanks are due to Almighty God , who inspired me to bring forth to light this thesis .

I wish to express my deep thanks to Head of Department of Mathematics , Faculty of Science , Tanta University for his encouragement and endless help .

I'd like to say that words are not enough and incapable to express exactly my sincer thanks to my supervisors **Prof . Dr.M.E.Abd El-Monsef**, professor of Pure Mathematics and Vice Dean Faculty of Science, Tanta University who introduced me to the world of topologists for suggesting the problem of this research , for his continuous encouragement , kindhearted help and valuable discussions .

My gartitude and deep thanks to **Dr. S.El Assar** Assistant profosser of Mathmatics, Faculty of Science for sincere supervision. Also my gratitude and deep thanks to **Dr. E.F. Lashien** Assistant profossor of Mathematics, Faculty of Ingnering for her everlasting help.

My thanks are extended to all members of the staff of Mathematics Department, Faculty of Science , Tanta University, especially . **prof. Dr. M.K.Gabr, Dr. A.M.Koza** for their encouragement. Also my thanks are due to **Dr. A.A. Nasef** for his continuous help .

It is pleasure to give my thanks to all collegues of Mashhour Topological School.

I am deeply indebted and heartly grateful to my mother and my husband who stood beside me and gave me hope and encouragement.

Fainally , I am grateful to **Mrs. M.Ameen** for typing this thesis.

# CONTENTS

Page

# PREFACE

The concept of bitopological space was initiated by J.C.Kelly [ 62 ] in 1963 . After the publication of Kelly's paper many toplogiests have shown interest in the study of such spaces . The problem of defining bitopological compactness has been considered by several authors in particular, Fletcher, Hoyle and Patty [38] , Kim [64] , Birsan [22] ,Swart [112] and Reilly [103].

In 1982 , Munshi and Bassan [87] have introduced the concept of S-compact spaces . Mashhour , et . al. have defined the concept of pairwise S - compactness [82] . In other paper Mashhour , et .al .have study the notion of pairwise - S - closed spaces [ 97] . Swart [112 ] , has introduced the concept of pairwise - C - compactness . Reilly [102] , has introduced the concept of pairwise - Lindelöf spaces . Abd El-Monsef , et. al. have introduced the concepts of pairwise - S - Lindelöf spaces , pairwise - M - S - Lindelöf spaces , pairwise - S - C compact spaces , pairwise - S - closed spaces and pairwise - S - closed sets [12] .

The genesis of concept of ideals in general topological spaces is treated in the classic text by Kuratawski in 1966 [66] .Also , several authors have interested in this line of study and therefore some sorts of ideals arise as one goes further in mathematics such as the ideal of finite subsets of X , the ideal of nowhere dense sets and the ideal of meager sets . Different types of operators in terms of ideals , compactness via an ideal , sets , functions and other concepts were investigated by many topologists . The concept of the set operator $( )^{\bullet} : P(X) \to P(X)$ had been introduced by

Vaidyanathaswamy in 1945 . In 1990 , Hamlett , Rose and Jankovic have studied many properties of set operator ( )* in [ 46 , 47 , 48 , 49 ] .

The concept of compactness with respect to an ideal was first defined by Newcomb in 1967 [91] and has been also studied by Rancin in 1972 [101] . Compactness with respect to an ideal (I-compactness ) has been studied extensively in [ 46 , 59 , 101 ] .

This thesis , which consists of four chapters is devoted to introduce and study new concepts in bitopological compactness via ideals and construct some types of sets , functions and operators in terms of ideals .

**In chapter I** , we give an exposition of some needed definitions and preliminaries to be used throughout this thesis . Moreover we introduce some results concerning fundamental concepts in this work . The aim of §1.1, §1.2 and §1.3 , is to introduce a historical background on ideals and on bitopological spaces , some basic concepts on ideals , some near open sets , some types of spaces ,some kinds of functions are investegated . Also , we give some main results which will be used in this work . §1.4 . §1.5 and §1.6 are devoted to introduce preliminaries and fundamental properties about local functions , compactness in bitopological spaces and compactness with respect to ideals .

**In chapter II** , we give additional characterizations and properties of the set operator $( )^*$ . Some types of compactness in bitopological spaces via ideals and some basic concepts on near - I - open sets are studied .

§2.1 is devoted to introduce and investigate the set operator $(\ )^*$, pairwise - I - Hausdorff. In §2.2, we define and study pairwise - I - compact spaces, the images of these spaces under pairwise - continuous functions are investigated. § 2.3 deals with a new type of compactness via ideal (pairwise $-\ I^*-$compact spaces ) and we obtain some new properties and characterizations of it. Also we define some new types of functions and the images of these spaces under pairwise $-\ I^*-$ continuous, pairwise $-\ I^*-$ open, pairwise$-\ E^*-$contiuous, pairwice $-\ E^*-$open functions are investigated. In § 2.4. we initiate and study a new concept called pairwise - I - S -compact spaces. Some characterizations and properties of these concepts are obtained. Our task in §2.5 is to give a further investigation for the concepts of pairwise $-\ I^*-S-$ compact spaces and some of characterizations and properties of these concepts are given.

Some results of Chapter II are written as a research paper intitled [Some new types of bicompactness ] and it is presented in " Third Internatial Conference of Mathematics on Geometry, Topology and Applications" Cairo 1997.

**In chapter III**, we construct a new type of closeness which we call pairwise - I - S - closed spaces § 3.1 and we give some basic concepts of it. In §3.2. we introduce some properties and characterizations of the new notion pairwise $-\ I^*-S-$ closed spaces. In §3.3. we initiate and study a new concept called pairwise - I - strongly compact spaces. Some