FACTORS IN THE PREDICTION OF BLAST INDUCED GROUND VIBRATIONS

By

Mostafa Fathy Amin

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Mining Engineering

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Under the Supervision of

Prof. Dr.	Prof. Dr.
Yehia Saad-Eldin Mohamed	Elseman Ibrahim Abdel-Rasoul
Professor of Mining Engineering Department of Mining, Petroleum and Metallurgy Faculty of Engineering, Cairo University	Professor of Mining Engineering Department of Mining and Metallurgy Faculty of Engineering, Assiut University

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Approved by the
Examining Committee

____
Prof. Dr. Yehia Saad-ELdin Mohamed, Thesis Main Advisor

____
Prof. Dr. Elseman Ibrahim Abdel-Rasoul, Thesis Advisor

____
Prof. Dr. Mostafa Tantawy Mohamed, External Examiner

____
Prof. Dr. Hassan Fahmy Imam, Internal Examiner

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT
2017

Engineer's Name:	Mostafa Fathy Amin Abd Elsalam
Date of Birth:	17/ 8 / 1989
Nationality:	Egyptian
E-mail:	mostafa.college11@yahoo.com
Phone:	01144212921
Address:	3 Hassan Moussa St.,Faisal St.,Giza
Registration Date:	1 / 10 / 2011
Awarding Date:	…./…./……..
Degree:	Master of Science
Department:	Mining Engineering
Supervisors:	Prof. Dr. Yehia Saad-Eldin Mohamed Prof. Dr. Elseman Ibrahim Abdel-Rasoul
Examiners:	Prof. Dr. Yehia Saad-ELdin Mohamed (Thesis main advisor) Prof. Dr. Elseman Ibrahim Abdel-Rasoul (Thesis advisor) Prof. Dr. Mostafa Tantawy Mohamed (External examiner) Prof. Dr. Hassan Fahmy Imam (Internal examiner)

Title of Thesis:

Factors in the Prediction of Blast induced ground vibrations

Key Words:
Rock blasting; Blast induced ground vibrations; Prediction; Empirical models; Site constants.

Summary:

In this thesis, different empirical models for the prediction of blast induced ground vibrations have been investigated using data previously collected from five different limestone quarries in Egypt. Based on this data, the objectives were to determine the prediction model that best describes the ground vibrations considering the results of previous studies at these quarries. Additionally, the effects of blast design parameters and intact rock properties on the propagation of ground vibrations (i.e. attenuation site constants (k, n)) have been investigated.

The results have shown an improvement in the prediction of ground vibrations at some sites. Based on the results, an allowable maximum charge per delay has been suggested based on the measured vibration frequencies and the available vibration standards.

It was also found that site constant (k) is more affected by intact rock properties than site constant (n). While the blast design parameters have varying effects on the site constants (k, n).

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Dedication

To my Beloved Mother.

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List of Symbols

Symbol	**Meaning**
σ	Normal Stress
σ_T	Transmitted stress
σ_d	Detonation pressure or Peak charge pressure
ε	Normal strain
E	Modulus of Elasticity
ρ_r	Density of rock
ρ_e	Density of explosive
C	Propagation velocity
λ	Vibration Wavelength
T	Vibration Period
f	Vibration frequency, where $f = \frac{1}{T}$
f_N	Natural frequency of a building
ω	Vibration angular frequency
A	Particle Amplitude
v	Particle velocity
v_0	The uncorrected vertical peak particle velocity
v_v	Vertical particle velocity
a	Particle acceleration
$x(t)$	Particle displacement at any time
φ	Phase angle of the vibration waves
θ	Angle between successively detonated holes and the position of the monitor