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# Granular Computing via Covering Models

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# Chapter 1

# Introduction

Granular computing "GrC" is an emerging conceptual and computing model of information processing. Granular computing was first introduced in 1997 by T.Y Lin to label a new field of multi-disciplinary study [23, 24]. From that time, there is a rapid development and an accelerated interest in this topic [5, 16, 25, 26, 27, 30, 43, 50, 51, 52, 55, 56, 60, 66, 67, 68, 90]. The purpose of this chapter is to give a short survey of the meaning and the importance of Granular computing. It contains methods and models of Granular computing. The results enhance our understanding of granular computing.

In this chapter we give an overview on granular computing and some methods of granular computing. In Section 1.1, we try to answer on the questions: what is granular computing (Grc)?, why do we study Grc?, what is new?,

what are basic issues of Grc?. Fuzzy set theory is given in Section 1.2. In Section 1.3 we present the concepts of rough set theory. Some fundamental notions of topology are given in the last section.

## 1.1 Granular computing (GrC)

We are living in a world which is undergoing profound changes brought by rapid advances in science and technology.

Among such changes , the most visible are those that relate to what is popularly referred to as the information revolution. The faces of this revolution are all around us: the e-mail, the world wide web, the cellular phone; the fax; the desktop computer and among many others.

Linked to the information revolution is another revolution- the intelligent systems revolution. The information and intelligent systems revolutions are in a symbiotic relationship. Intelligence requires information and vice-versa. The confluence of intelligent systems and information systems leads to intelligent information systems. In this sense, the union of information systems, intelligent systems and intelligent information systems constitutes what might be referred to as information/ intelligent systems, or I/IS for

short. In coming years, the design, construction and utilization of information/ intelligent systems will become the primary focus of science and technology, and I/IS systems will become a dominant presence in our daily lives. When we take a closer look at information/intelligent systems what we see is the increasingly important role of Granular computing (GrC) in their conception, design and utilization.

Existing studies of granular computing typically concentrate on concrete models and computational methods in particular contexts. They unfortunately only reflect specific aspects of granular computing. In fact, there does not exist a formal, precise, commonly agreed, and uncontroversial definition of what is granular computing, nor there is a unified model. Consequently, the potential applicability and usefulness of granular computing are not well perceived and appreciated. Since concrete models and methods of granular computing have been extensively studied by many authors, we focus on a high, conceptual level investigation in an attempt to address some of the fundamental issues. The main objective is to discuss some important perspectives of granular computing, based on our initial research on establishing a holistic, whole, and integrated view of granular computing [69, 70, 71, 72, 73, 74, 75, 76, 77, 78].

The discussion is divided into two parts. In the first part, we study perspectives of granular computing by trying to answer, at least partially, the following questions:

What is granular computing?

Why do we study granular computing?

What is new and different in granular computing?

What are basic issues of granular computing?

An examination of these questions enables us to derive, in the second part, overview of two Granular computing models (GrC).

### 1.1.1 What is granular computing ?

The following quotations from Zadeh may help us in understanding the scope of, and reasons for, granular computing (GrC):

> "Granulation of an object $A$ leads to a collections of granules of $A$, with a granule being a clump of points (objects) drawn together by indistinguishability, similarity, proximity or functionality" [83].

> "The theory of fuzzy information granulation (TFIG) is inspired by the ways in which humans granu-

late information and reason with it" [83].

"GrC is a superset of the theory of fuzzy information granulation, rough set theory and interval computations, and is a subset of granular mathematics" [83].

It is clear that an underlying idea of granular computing is the use of groups, classes, or clusters of elements called granules [83, 84]. Although extensive work has been done on granular computing, it still might be difficult to give a precise definition. In stead, Granular computing (GrC) may be regarded as a label of theories, methodologies, techniques and tools that make use of granules, i.e., groups, classes, or clusters of a universe, in the process of problem solving.

Several fields contribute significantly to the study of granular computing. Many researchers in granular computing community formulate the problem based on theories and models of computational intelligence [5, 28, 25, 24, 26, 43, 51, 52, 55, 56, 79, 72, 68, 87]. In 1979, Zadeh first introduced the notion of information granulation and suggested that fuzzy set theory may find potential applications in this respect [84]. In 1982, Pawlak proposed the theory of

rough sets [44, 45], which in fact provides a concrete example of granular computing. To some extent, rough set theory makes more people realize the importance of the notion of granulation. In 1997, Zadeh revisited information granulation [83], which led to a renewed interest. In the same year, Lin suggested the term granular computing to label the new and growing research filed [23, 24]. Moreover, Lin proposed to use neighborhood systems for the formulation of granular computing [28, 25, 24, 26, 69], which is an extension of the partition-based rough set theory. Yao interpreted granular computing in a wide context based on the principles and ideas from other fields of computer science [73, 76]. The ideas of granular computing have been investigated in artificial intelligence through the notions of granularity and abstraction. Hobbs proposed a theory of granularity [15], which is similar to the theory of rough sets in terms of formulation. The theory indeed captures some of the essential features of granular computing. That is, we perceive and represent the world under various grain sizes, and abstract only those things that serve our present interests. The ability to conceptualize the world at different granularities and to switch among these granularities is fundamental to our intelligence and flexibility. This enables us to map the complexities of real world into computationally tractable simpler theories.

We view granular computing as a multi-disciplinary study with the objective to investigate and model the family of granule-oriented problem solving methods and information processing paradigms [70, 75]. It is a study of a general theory of problem solving based on different levels of granularity and detail [70, 75, 88, 89].

Our view is based on Granular computing (GrC) may be regarded as a label of theories, methodologies, techniques and tools that make use of granules, i.e., groups, classes, or clusters of a universe, in the process of problem solving. Granular computing, therefore, focuses on everyday and commonly used concepts and notions, such as granule, granulated view, granularity, and hierarchy. The notions of granular computing may be interpreted in terms of abstraction, generalization, clustering, levels of abstraction, levels of detail, and so on in various domains.

### 1.1.2 Why do we study granular computing

There are many reasons for the study of granular computing. The previous discussion provides some motivations. They stem mainly from the use of levels of granularity. The following list summarizes and reiterates some of the points:

1) **Truthful representation of the real world**. Many natural, social, and artificial systems are organized into levels [54]. Granular computing provides true and natural representations of such systems. Through the multiple level representation, one can obtain a full understanding of a system.

2) **Consistent with human thinking and problem solving**. From a philosophical and theoretical point of view, many authors argued that information granulation is very essential to human problem solving and hence has a very significant impact on the design and implementation of intelligent systems. Human problem solving is based crucially on levels of granularity and change between granularities [15, 83]. Zadeh [83] identified three basic concepts that underlie human cognition,namely, granulation, organization and causation. "Granulation involves decomposition of whole into parts, organization involves integration of parts into whole and causation involves association of causes and effects."

3) **Simplification of problems**. A multiple level representation shows the orderness, the control, and the organization of a complex system or a complex problem. From a more practical point of view, the neces-

sity of information granulation and simplicity derived from information granulation in problem solving are perhaps some of the main reasons. In many situations, when a problem involves incomplete, uncertain or vague information, it may be difficult to differentiate distinct elements and one is forced to consider granules.

4) **Economic and low cost solutions**. By considering the same problem at different levels of granularity, we ignore some details. This in turn may lead to approximate and inaccurate solutions [83]. The lack of information may only allow us to define granules rather than individuals. In some situations, although detailed information may be available, it may be sufficient to use granules in order to have an efficient and practical solution. In fact, very precise solutions may not be required at all for many practical problems. It may also happen that the acquisition of precise information is too costly and coarse-grained information reduces cost.

In summary, the benefits of granular computing is evident from its basic guiding principle, which is given by Zadeh concisely as to "exploit the tolerance for imprecision, uncertainty and partial truth to achieve tractability,