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**Electromagnetic Diffraction through Sub-Wavelength
Apertures**

A Thesis submitted by

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This dissertation is submitted to Ain Shams University for the degree of Doctor of Philosophy in Engineering Physics.

The work included in this thesis was carried out by the author at the Department of Engineering Physics and Mathematics, Faculty of Engineering, Ain Shams University, Cairo, Egypt.

No part of this thesis was submitted for a degree or a qualification at any other university or institution.

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Summary

The problem of investigating the behavior of electromagnetic waves, especially in the optical region, when diffracting through sub wavelength dimensions has recently become a very active field of research due to the presence of this problem in different applications including photolithography and several other nanoscale applications. With the increasing complexity of the structures based on sub wavelength features, the need for fast and efficient numerical techniques to be used for the simulation of such structures has increased.

Common techniques used for simulating structures with dimensions not in the order of the wavelength suffer from very large memory requirements and simulation time, making their use in sub wavelength problems not an optimum choice. In this thesis, a numerical technique is proposed for simulating the behavior of electromagnetic waves in such miniaturized dimensions. After a detailed explanation of the proposed technique, it is applied on different structures and the results are compared to published simulation and experimental results in order to verify the validity of the technique.

The thesis is structured as follows: An introduction for the thesis includes a brief survey on the content of the thesis.

In Chapter 1, different applications for the problem of diffracting electromagnetic waves through sub wavelength apertures are illustrated.

The chapter starts with a presentation of the theoretical background of the diffraction problem, followed by a demonstration for the diffraction phenomena in photolithography and in modern nanoscale structures.

In Chapter 2, a survey is done on different numerical techniques that are used in the simulation of the electromagnetic diffraction problem. Partial coherent sources and their effect on the simulation process is also investigated in this chapter.

In Chapter 3, a modal technique is proposed for the numerical solution of sub wavelength problems. The technique is derived mathematically and a detailed discussion on the applicability of the technique to the problems under investigation is given.

In Chapter 4, the proposed technique is applied on three different structures. The first structure consists of a two dimensional square aperture where the results are compared to published simulation results. The second structure is a one dimensional planar lens structure consisting of sub wavelength slits of different widths. The results are compared to published experimental results. The third structure is also a planar lens where slit widths are chosen such that a Bessel beam like wavefront is obtained in order to achieve large focal lengths.

Conclusions based on the simulation results for the different structures and the suitability of the proposed technique are then given together with possible improvements and further investigation.

List of Abbreviations

ADE	Auxiliary Differential Equation
BC	Boundary Conditions
CD	Critical Dimension
CMOS	Complementary Metal Oxide Silicon
DOF	Depth of Focus
FDTD	Finite Difference Time Domain
FDFD	Finite Difference Frequency Domain
FIB	Focused Ion Beam
FMM	Fourier Modal Method
MEMS	Micro Electro-Mechanical Systems
NA	Numerical Aperture
ODE	Ordinary Differential Equation
OPC	Optical Proximity Correction
PEC	Perfect Electric Conductor
PML	Perfectly Matched Layer
PSF	Point Spread Function
PSM	Phase Shift Mask
RET	Resolution Enhancement Technique
RC	Recursive Convolution
RCWA	Rigorous Coupled Wave Algorithm
RSM	Radiation Spectrum Method
SPP	Surface Plasmon Polariton
SRR	Split Ring Resonator
TLM	Transmission Line Method
UV	Ultra Violet
VIEM	Volume Integral Equation Method

List of Symbols

k	wave vector
λ	wavelength
\hbar	reduced Planck's constant
e	charge of an electron
m_e	mass of an electron
n	index of refraction
ϵ	permittivity
ϵ_0	permittivity of free space
ϵ_r	relative permittivity
μ	magnetic permeability
μ_0	magnetic permeability of free space
β	propagation constant
D	electric flux density
ρ	volume charge density
B	magnetic flux density
E	electric field intensity
H	magnetic field intensity
J	electric current density
σ	electric conductivity
ω	angular frequency
f	frequency
χ	electric susceptibility

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