

THEORETICAL STUDY OF POSITRON - EXCITED ALKALI ATOM COLLISIONS

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By

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CONTENTS

	Page
Acknowledgement	
Abstract	
Chapter 1: Introduction	
1.1 The importance of scattering	1
1.1.1 Types of collisions	2
1.1.2 Definition of channel	2
1.2 The difference between electron and positron scattering	3
1.3 Positronium atom	6
1.4 Alkali metal atoms	7
1.5 Positron-alkali atom scattering	8
1.6 Survey of the previous work in positron alkali atom collisions	9
Chapter 2: Theoretical treatment of the two - channel problem of positron-excited lithium scattering.	
2.1 The bound-state wave functions	14
2.2 The total energy	15
2.3 The total Hamiltonian	15
2.4 Analyses of the two-channel scattering problem	18
2.4.1 Analysis of the first coupled integro - differential equation (eq. (2.18))	18
2.4.2 Analysis of the second coupled integro - differential equation (eq. (2.19))	21
2.5 Solution of the two-channel coupled integro - differential equations	23
2.6 Reactance and transition matrices	26
2.7 The partial and total cross-sections	27

CONTENTS (Cont...)

	Page
Chapter 3: Results and discussion	28
Summary	63
Appendix I: Analyses of the potentials	
1.1 The direct part of the core potential	66
1.2 The exchange part of the core potential	69
1.3 The static potential of the first channel $U_{st}^1(x)$	71
1.4 The static potential of the second channel $U_{st}^2(\sigma)$	80
1.5 The polarization potential of the second channel $V_{pol}^{Ps}(\sigma)$	81
1.6 Gauss quadrature formula	82
Appendix II: The coupling kernals of the two channel problem.	
II.1 The coupling kernel of the first channel (eq. (2.29)	84
II.2 The coupling kernel of the second channel (eq. (2.37)	88
II.3 Numerical treatment of the kernels	90
References	95
Arabic Summary	—

LIST OF TABLES

Tab. No.	Title	Page No.
Table 3.1	Variation of the elements of the reactance matrix R with the number of iterations, at $k_1^2 = 0.5$ eV and Integration Range (IR) = 48 a.u., for the S-, P- and D- matrix waves	29
Table 3.2	Variation of the partial elastic cross sections σ_{11} (in πa_0^2) of positron-excited lithium ($Li^*(2p)$) scattering with the increase of the integration range IR (in a.u.)	30
Table 3.3	Variation of the partial positronium formation cross sections σ_{12} (in πa_0^2) of positron-excited lithium ($Li^*(2p)$) scattering with the increase of the integration range IR (in a.u.)	31
Table 3.4	Variation of the partial rearrangement cross sections σ_{21} (in πa_0^2) of positronium – lithium ion (Li^+) scattering with the increase of the integration range IR (in a.u.)	32
Table 3.5	Variation of the partial elastic cross sections (in πa_0^2) of positronium – lithium ion σ_{22} (Li^+) scattering with the increase of the integration range IR (in a.u.)	33
Table 3.6	Partial and total elastic cross sections σ_{11} (in πa_0^2) of positron-excited lithium ($Li^*(2p)$) scattering without polarization potential calculated by the coupled-static approximation. Integration range IR = 48 a.u. and the number of iterations = 20	36
Table 3.7	Partial and total positronium formation cross sections σ_{12} (in πa_0^2) of positron-excited lithium ($Li^*(2p)$) scattering without polarization potential calculated by the coupled-static approximation. Integration range IR = 48 a.u. and the number of iterations = 20	37

LIST OF TABLES (Cont...)

Tab. No.	Title	Page No.
Table 3.8	Partial and total rearrangement cross sections σ_{21} (in πa_0^2) of positronium- lithium ion (Li^+) scattering without polarization potential calculated by the coupled-static approximation. Integration range $\text{IR} = 48 a.u.$ and the number of iterations = 20.....	38
Table 3.9	Partial and total elastic cross sections σ_{22} (in πa_0^2) of positronium-lithium ion (Li^+) scattering without polarization potential calculated by the coupled-static approximation. Integration range $\text{IR} = 48 a.u.$ and the number of iterations = 20.....	39
Table 3.10	Partial and total elastic cross sections σ_{11}^P (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering with polarization potential calculated by the coupled-static approximation. Integration range $\text{IR} = 48 a.u.$ and the number of iterations = 20.....	40
Table 3.11	Partial and total positronium formation cross sections σ_{12}^P (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering with polarization potential calculated by the coupled-static approximation. Integration range $\text{IR} = 48 a.u.$ and the number of iterations = 20.....	41
Table 3.12	Partial and total rearrangement cross sections σ_{21}^P (in πa_0^2) of positronium -lithium ion (Li^+) scattering with polarization potential calculated by the coupled-static approximation. Integration range $\text{IR} = 48 a.u.$ and the number of iterations = 20.....	42

LIST OF TABLES (Cont...)

Tab. No.	Title	Page No.
Table 3.13	Partial and total elastic cross sections σ_{22}^p (in πa_0^2) of positronium-lithium ion (Li^+) scattering with polarization potential calculated by the coupled-static approximation. Integration range IR = 48 <i>a.u.</i> and the number of iterations = 20.	44
Table 3.14	Partial and total elastic cross sections σ_{11}^{pl} (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering with polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range IR = 48 <i>a.u.</i> and the number of iterations = 20.....	45
Table 3.15	Partial and total positronium formation cross sections σ_{12}^{pl} (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering with polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range IR = 48 <i>a.u.</i> and the number of iterations = 20.....	46
Table 3.16	Partial and total rearrangement cross sections σ_{21}^{pl} (in πa_0^2) of positronium- lithium ion (Li^+) scattering with polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range IR = 48 <i>a.u.</i> and the number of iterations = 20.	47
Table 3.17	Partial and total elastic cross sections σ_{22}^{pl} (in πa_0^2) of positronium- lithium ion (Li^+) scattering with polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range IR = 48 <i>a.u.</i> and the number of iterations = 20.....	48

LIST OF TABLES (Cont...)

Tab. No.	Title	Page No.
Table 3.18	Partial and total elastic cross sections σ_{11}^I (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering without polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range $\text{IR} = 48 \text{ a.u.}$ and the number of iterations = 20.	49
Table 3.19	Partial and total positronium formation cross sections σ_{12}^I (in πa_0^2) of positron-excited lithium ($\text{Li}^*(2p)$) scattering without polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range $\text{IR} = 48 \text{ a.u.}$ and the number of iterations = 20.	50
Table 3.20	Partial and total rearrangement cross sections σ_{21}^I (in πa_0^2) of positronium- lithium ion (Li^+) scattering without polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range $\text{IR} = 48 \text{ a.u.}$ and the number of iterations = 20.	51
Table 3.21	Partial and total elastic cross sections σ_{22}^I (in πa_0^2) of positronium-lithium ion (Li^+) scattering without polarization potential calculated by the coupled-static approximation (intermediate energy region). Integration range $\text{IR} = 48 \text{ a.u.}$ and the number of iterations = 20.	52
Table 3.22	Comparison between various total positronium formation cross sections (in πa_0^2) of positron-lithium scattering determined by different authors.	59
Table II-1	Arguments of $G_l^{(j)}$ at different values of j	94

LIST OF FIGURES (Cont...)

Fig. No.	Title	Page No.
Fig 2.1	Configuration space of positron- excited lithium scattering.....	13
Fig. 3.1	Comparison between our calculated total elastic (σ_{11}) and total Ps formation cross sections (σ_{12}) of e^+ - $Li^*(2p)$ scattering without polarization potential.	54
Fig 3.2	Comparison between our calculated total rearrangement (reversal of the positronium formation) cross sections (σ_{21}) and total elastic cross sections (σ_{22}) of Ps - Li^+ scattering without polarization potential.	54
Fig 3.3	Same as Fig 3.1 but with polarization potential.	55
Fig 3.4	Same as Fig 3.2 but with polarization potential.	55
Fig 3.5	Same as Fig 3.1 but at intermediate energy region.	56
Fig 3.6	Same as Fig 3.2 but at intermediate energy region.	56
Fig 3.7	Same as Fig 3.3 but at intermediate energy region.	57
Fig 3.8	Same as Fig 3.4 but at intermediate energy region.	57
Fig 3.9	Shows the effect of polarization potential on the total elastic cross sections of e^+ - $Li^*(2p)$ scattering.....	60
Fig 3.10	Shows the effect of polarization potential on the total Ps formation cross sections of e^+ - $Li^*(2p)$ scattering.....	60
Fig 3.11	Shows the effect of polarization potential on the total rearrangement (reversal of the Ps formation) cross sections of Ps - Li^+ scattering.	61

LIST OF FIGURES (Cont...)

Fig. No.	Title	Page No.
Fig 3.12	Shows the effect of polarization potential on the total elastic cross sections of Ps - Li ⁺ scattering.....	61
Fig 3.13	Comparison between various total positronium formation cross sections (in πa_0^2) of e ⁺ - Li scattering determined by different authors.....	62

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ABSTRACT

The inelastic scattering of positrons by excited lithium alkali atoms $\text{Li}^*(2p)$ have been investigated within the frame work of the coupled – static and frozen – core approximations with the assumption that the elastic and rearrangement channels are open. In the present work, a rather complicated computer code is developed based on the coupled – static, frozen – core and Green's function partial wave expansion technique. The partial and total elastic and positronium (Ps) formation cross sections of $e^+ - \text{Li}^*(2p)$ are calculated through a wide range of incident energy of positrons ranging from 0.3 eV to 1000 eV. Also, we have calculated the partial and total elastic and rearrangement (reversal of the Ps formation) cross sections of $\text{Ps} - \text{Li}^+$ collisions through the low, intermediate and high energy regions.

The effect of polarization potential of the Ps atom is taken into our consideration. The total cross sections which corresponding to twelve partial cross sections (calculated at twelve values of the total angular momentu $l = 0$ to $l = 11$) are calculated for each channel. Our calculated total positronium formation cross sections are compared with experimental results and those calculated by other authors. The present calculations encourage the experimental physicists to carry out positron – lithium experiments by taking the excited lithium target into accounts in order to obtain more positronium especially in the low and intermediate energy regions.

Keywords: Positrons; Positronium formation; Alkali atoms; Collisions; Inelastic scattering; Cross-sections; Lithium; Polarization potential.

Chapter 1

INTRODUCTION



1.1 The importance of scattering

Atomic scattering (collision) is a very wide topic. The field of atomic scattering has a venerable history, since it was central to the development of quantum mechanics, and nuclear physics. Atomic scattering are the basic means for probing the atomic structure of matter. Almost every thing we know about nuclei and elementary particles has been discovered in scattering experiments, from Rutherford's surprise at finding that atoms have their mass and positive charge concentrated in almost point - like nuclei, to the more recent discoveries, on a far smaller length scale, that protons and neutrons are themselves made up of apparently point - like quarks. More generally, the methods that we have to probe the properties of condensed matter systems rely fundamental on the notion of scattering. Study of scattering processes is the main source of information about strong, electromagnetic and weak interactions [Burke].

The interaction of antimatter with matter is an interesting and active field of study. Positron interactions with matter play important roles in many physical processes of interest. Examples include the origin of astrophysical sources of annihilation radiation, the use of positron in medicine (e.g, positron emission tomography); the characterization of materials; and the formation of antihydrogen, which is the simplest of stable, neutral antimatter [Marler].