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HOMOLOGY AND THE DEFORMATION RETRACT OF SOME MANIFOLDS AND THEIR FOLDINGS

A Thesis

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by

Mohammad Hamed Mostafa Mohammad

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CONTENTS

PREFACE

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A fundamental problem of topology is that of determining, for two topological spaces, whether or not they are homeomorphic (i.e., topologically equivalent). To show that two spaces are homeomorphic, one needs to construct a continuous bijective map, with continuous inverse, mapping one space to the other. To show that two spaces are not homeomorphic involves showing that such a map does not exist. To do that is often harder.

As an alegbraic means to tackle such problems, Betti associated with each topological space a certain sequence of abelian groups called its homology groups. It was later proved that homeomorphic spaces have isomorphic homology groups; that is, two spaces with homology groups which are not isomorphic are necessarily not homeomorphic.

As the title of this thesis suggests, we are concerned with the study of the effects of some types of deformation retractions, foldings, and unfoldings on the homology groups of some types of real and complex manifolds; that is, whether or not the original manifold maintains its homology groups (up to isomorphism).

Chapter 1 is of an introductory nature. It sets the stage for the coming chapters by accommodating basic topological and geometric concepts and basic results needed in the thesis.

In chapter 2, we discuss relations between the homology groups of some (real) manifolds and those of their retracts. We also establish theorems concerning relations between the homology groups of these manifolds and the homology groups of their deformation retracts before

and after topological folding. The results of this chapter were accepted for publication in the Bulletin of Calcutta Mathematical Society (cf. [18]).

In chapter 3, we discuss deformation retracts of some types of complex manifolds (complex projective space). Then we study relations between the homology groups of these complex manifolds and the homology groups of their deformation retracts. Also studied are the effects of some foldings on the homology groups of the complex projective space. The results of this chapter were submitted for publication in the Journal of Le Mathematiche Dipartmento Di Matematica Citta Universitaria Viale A Doria, 6-I 95125, Catania, Italy.

In chapter 4, we introduce the concept of an unfolding of a manifold and then investigate the effect of the unfolding on the homology groups of the original manifold. Also studied are the effects of retractions and some types of unfoldings on the homology groups of complete graphs. The results of this chapter were submitted for publication in the Bulletin of the Malaysian Mathematical Society.

Mohammad Hamed Mostafa

CHAPTER 1

CHAPTER 1

BASIC CONCEPTS

For convenience, ease and accuracy of reference, basic topological and geometric concepts and results relevant to this work are cited in this introductory chapter. We adapted these concepts to fit coherently into the framework of this thesis.

1.1. Continuity and Homemorphisms

Let (X,τ) and (Y,η) be topological spaces. For each point $p \in X$, we denote the set $\{N \subseteq X : N \text{ is a neighbourhood of } p\}$ by "ν_p", the set $\nu_p \cap \tau$ by "o_p", and the set $\{V \subseteq X : X \setminus V \in \tau\}$ by "τ^c".

A function $f : X \to Y$ is said to be <u>continuous at a point</u> $p \in X$ if any of the following equivalent conditions is met.

(i) $(\forall V \in o_{f(p)})(\exists U \in o_p)(f[U] \subset V)$.

(ii) $(\forall N \in \nu_{f(p)})(\exists M \in \nu_p)(f[M] \subseteq N)$.

(iii) $(\forall N \in \nu_{f(p)})(\exists M \in \nu_p)(M \subseteq f^{-1}[N])$.

(iv) $(\forall V \subset o_{f(p)})(\exists U \subset o_p)(U \subseteq f^{-1}[V])$.

(v) $(\forall N \in \nu_{f(p)})(f^{-1}[N] \in \nu_p)$.

(vi) $(\forall V \in o_{f(p)})(f^{-1}[V] \in \nu_p)$.