Ain Shams University

Some New Valuations of Graphs

Thesis by

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Abstract

In this the, we study : fibonacci divisor cordial labeling , prime cordial labeling ,near mean cordial labeling ,square difference labeling, and we prove some theorems. Also we give some of their families .

Keywords

Graph labeling, fibonacci divisor cordial labeling, prime cordial labeling, near mean cordial labeling, square difference labeling.

Summary

Graph theory is one of the important branches of Graph Theory and became a principal tool in many applications on different sciences and technologies.

In this thesis we study four main types of graph labeling and results for graphs of these types .We divide the work into five chapters:

Chapter one: We introduce some basic definitions and theorems in number theory and graph theory which we need afterwards.

Chapter two: We deal with "Fibonacci divisor cordial graphs". We first define the labeling. Then we give the maximum number of edges of a graph which is Fibonacci divisor cordial .We prove that K_n is not a Fibonacci divisor cordial graph if $n \neq 9$.We prove that the graphs, the splitting graph $S'(B_{n,n}), K_{2,n}^{(2)}, < K_{1,n}^{(1)}, K_{1,n}^{(2)}, K_{1,n}^{(3)} >$, the degree splitting graph $DS(B_{n,n})$and the binary tree are Fibonacci divisor cordial graphs. Also we give a survey of all graphs of order ≤ 6 which are not Fibonacci divisor cordial labeling.

These results are accepted to be published in the International Journal of Mathematics Research (IJMR).

Chapter three: We deal with " Prime cordial graphs". We first define the labeling , we prove that the following graphs are prime cordial graphs: the disjoint of G and $K_{1,m}$, the splitting graph $S'(K_{2,n})$,Jelly fish graph, Jewel graph, the graph obtained by

duplicating a vertex v_k in the rim of the helm $H_n$and the graph obtained by fusing the vertex u_1 with $u_3$in a Helm graphH_n are prime cordial graphs .

These results are accepted to be published in Circulation in Computer Science and it was published on 20 May 2017.

Chapter four: We introduce the so-called "Near mean cordial graph". Here we prove that K_n is not a near mean cordial graph if $n \neq 3,4,6$. We prove that the graphs: the Book $K_{1,n} \times K_2$ when n is even,$(\overline{K_n} \cup P_n) + 2K_1$,the flower Fl_n, bistar $B_{n,n}$, $S'(K_{1,n})$, $B(3,2,m)$, B(4,3,m), Petresen graph and two Petresen graphs joined with an edge are mean cordial graphs. Also we give a survey of all graphs of order ≤ 7 which are not near mean cordial graphs.

These results are accepted to be published in "International Journal of Science and Research" (IJSR)

Chapter five: Here we deal with "Square difference graphs", we prove that for any square difference graph G, $f^*(e_i) \equiv 0,1,3(mod4)$, for every $e_i \in E(G)$ and if $f^*(e_i) \equiv 0(mod2)$ for every $e_i \in E(G)$, then G is a disconnected graph. We show the following graphs are square difference graphs: the graph obtained by joining the central vertex of $K_{1,m}$ to all vertices of $K_{1,n}$, and the central vertex of $K_{1,n}$ to all vertices of $K_{1,m}$, $K_{1,1,n}$,the butterfly graph $B_{n,m}$and a vertex switching of the wheel W_n. We give a survey of all graphs of order ≤ 7 which are not square difference graphs.

Contents

INTRODUCTION

Introduction

A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions.

Most graph labeling methods trace their origin to one introduced by Rosa [16] in 1967, or one given by Graham and Sloane [9] in 1980.Rosa [16] called a function f a β −valuation of a graph G with q edges if f is an injection from the vertices of G to the set {0,1,…,q} such that, when each edge xy is assigned the label $|f(x) - f(y)|$, the resulting edge label are distinct. Golomb[9] subsequently called such labelings graceful and this is now the popular term. Rosa introduced $\beta -$ valuations as well as a number of other labelings as tools for decomposing the complete graph into isomorphic subgraphs. In particular, $\beta -$ valuations originated as means of attacking the conjecture of Ringel [15] that K_{2n+1} can be decomposed into 2n+1subgraphs that are all isomorphic to a given tree with n edges. Although an unpublished result of Erdös says that most graphs are not graceful(cf.[10]), most graphs that have some sort of regularity of structure are graceful. Balakrishnan and Sampathkumar[3] have shown that every graph is a subgraph of a graceful graph. Rosa [16] has identified essentially three reasons why a graph fails to be graceful: (1) G has "too many vertices" and "not enough edges," (2) G "has too many edges," and (3) G "has the wrong parity." An infinite class of graphs that are not graceful for the second reason is given in [5]. As an example of third condition

Rosa [16]has shown that if every vertex has even degree and the number edges is congruent to 1 or 2(mod4) then the graph is not graceful. In particular, the cycles C_{4n+1} and C_{4n+2} are not graceful.

Harmonious graphs naturally arose in the study by Graham and Sloane [9] of modular vertices of additive bases problems stemming from error-correcting codes. They defined a graph G with q edges to be harmonious if there is an injection f from the vertices of G to the group of integers modulo q such that when each xy is assigned the label $f(x)+f(y)$(mod q), the resulting edge labels are distinct. When G is a tree, exactly one label may be used on two vertices. Analogous to the "parity" necessity condition for graceful graphs, Graham and Sloane proved that if a harmonious graph has an even number q of edges and the degree of every vertex is divisible by 2^k then q is divisible by 2^{k+1}. Liu and Zhang [11] have generalized this condition as follows: if a harmonious graph with q edges has degree sequence $d_1, d_2, \ldots, d_p$ then gcd($d_1, d_2, \ldots, d_p, q$) divides $q(q-1)/2$.

CHAPTER ONE

Background

CHAPTER ONE

Background

1.1 Some fundamentals in number theory [6]

Definition1.1.1: A positive integer n is said to be prime if it has no divisors other than 1 and n and composite otherwise.

Definition1.1.2: Let x be any real number then $\lfloor x \rfloor$ denote the greatest integer less than or equal to x.

1.2 Some fundamentals in graph theory

Definition1.2.1[10]: Let G(V,E) be a graph, where V(G) the vertex set and E(G) the edge set of G.

Definition1.2.2[10]: A vertex v is an end vertex of an edge e, v and e are said to be incident on each other.

Definition1.2.3[10]: Two edges are said to be adjacent if they are incident on a common vertex.

Definition1.2.4[10]: Two vertices u and v in V are said to be adjacent if they are the end vertices of same edge and in this case we write $uv \in E$.

Definition1.2.5[10]: The number of edges incident on a vertex v is called the degree of the vertex v.

Definition1.2.6[10]: Isolated vertices are vertices with zero degree.

Definition1.2.7[10]: A vertex of degree one is called a pendant vertex and the edge incident on it is called a pendant edge.

Example 1.2.8: In the following graph the vertex u is incident on the edge uv ,the two vertices u and v are adjacent, the edge wx is a pendant edge , the degree of the vertex w is 3, and the vertex y is an isolated vertex.

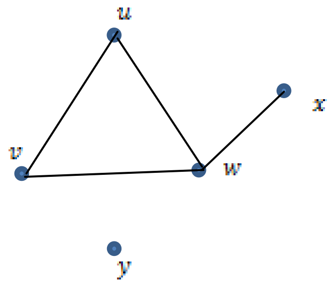


Figure 1.1

Definition1.2.9[10]: If all the vertices of a graph G have the same degree d, then G is called regular of degree d.

Example1.2.10: The following graph is a regular of degree 3.

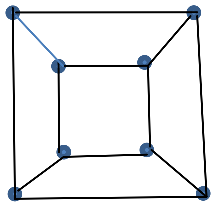


Figure 1.2

Definition1.2.11[10]: A graph H is said to be a subgraph of a graph G if $V(H) \subseteq V(G)$ and $E(H) \subseteq E(G)$. We write $H \subseteq G$ if h is a subgraph of G.

Example1.2.12:

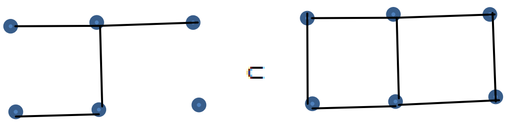
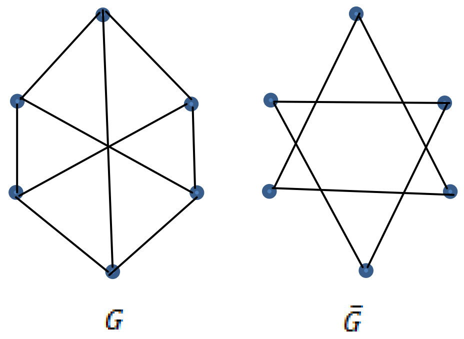


Figure 1.3

Definition1.2.13[10]: The complement $\bar{G}$ of G is a graph having vertex set $\mathrm{V(G)}$ and edge set $\{uv: u, v \in \mathrm{V(G)}, uv \notin E(G)\}$.

Example1.2.14: The following graph is a graph G and its complement $\bar{G}$ respectively.

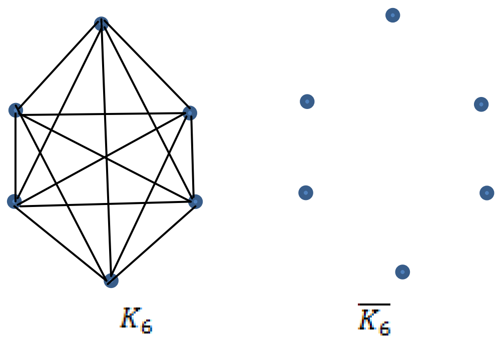


$\boldsymbol{Figure}\ \mathbf{1.4}$

Definition1.2.15[28]: A graph in which every pair of distinct vertices is adjacent is called a complete graph and we denote the complete graph on n vertices by K_n Realize that K_n has $\binom{n}{2} = \frac{n(n-1)}{2}$ edges.

Definition1.2.16[28]: A graph whose edge set is empty is called a null graph, we shall denote the null graph on n vertices by$\overline{K_n}$.

Example1.2.17: the following graphs are K_6 and $\overline{K_6}$ respectively.

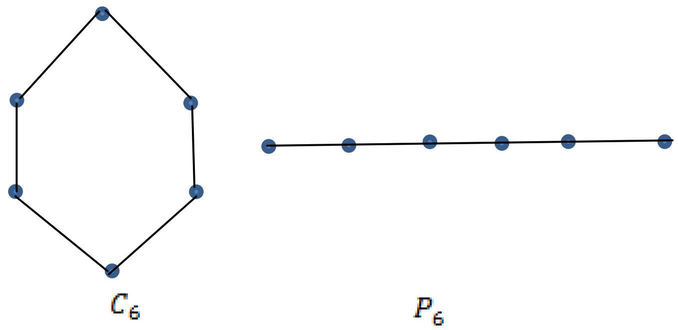


$\boldsymbol{Figure}\ \mathbf{1.5}$

Definition1.2.18[28]:A connected graph that is regular of order 2 is a cycle graph. We denote the cycle graph of order n by C_n.

Definition1.2.19[28]: The graph obtained from C_n by removing an edge is the path graph of order n, and denote it by P_n.

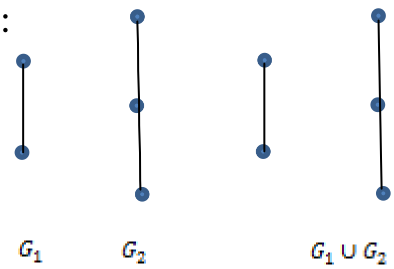
Example1.2.20: The following graphs are C_6 and P_6.



$\textit{Figure}\ 1.6$

Definition1.2.21[10]: Let G_1 and G_2 be two disjoint graphs (i.e., they have no vertices in common). The union $G_1 \cup G_2$ of G_1 and G_2 is the graph having vertex set $V(G_1) \cup V(G_2)$ and edge set $E(G_1) \cup E(G_2)$.

Example1.2.22:



$\textit{Figure}\ 1.7$

Definition1.2.23[10]: The join $G_1 + G_2$ of G_1 and G_2 is the graph having vertex set $V(G_1) \cup V(G_2)$ and edge set:

$$E(G_1) \cup E(G_2) \cup \{uv: u \in V(G_1), v \in V(G_2)\}.$$