Ain Shams University
Faculty of Engineering

Multi-Choice Linear Programming Models and It's Engineering Applications

A thesis submitted in partial fulfillment for the award of the

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Statement

This thesis is submitted as a partial fulfillment of Master of Science in Engineering Mathematics, Faculty of Engineering, Ain Shams University. The author carried out the work included in this thesis, and no part of it has been submitted for a degree or a qualification at any other scientific entity.

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Abstract

In some optimization problems, one parameter or more may have a multiple choices. These multi-choice values of the parameters are considered by the experts or decision makers. In this case, the optimization problem is called a multi- choice problem. To solve this type of problem, there is no direct method even in the case of a medium size problem involving multi- choice parameter is computationally expensive to obtain an optimal solution. Therefore, to develop a methodology that optimizes objective function and selects an appropriate multi- choice parameter, is one of the challenging problem in multi-choice programming problem. Keeping this in mind the core of the thesis is concentrated in building up solution procedures for multi- choice problem that can be implemented also to probabilistic and fuzzy programming problems involving multi- choice type parameters. Several parameters, namely cost coefficients, technical coefficients, resource limits may be taken into consideration when formulating a multi- choice programming problem. The thesis highlights the treatment of multi-choice problem considering the resource limits as multi- choice type parameters. It covers a detailed description of transformation techniques with the help of some variables known as binary variables to solve a multi- choice problem. It also describes the application of some numerical methods, namely interpolating polynomial methods for multi-choice parameters with the intention of avoiding the difficulties that arises during the usage of binary variables for transformation of multi-choice Linear programming (LP) problem to an equivalent multi- choice problem. Both transformation techniques are implemented for single objective multi- choice (LP) problems and both multi- objective, multi- choice (LP) problems.
The transformation techniques are implemented for single objective and multi-objective probabilistic programming problems, where the parameters in the right side are multi- choice type while the rest of the parameters in the probabilistic constraints are independent random variables which are distributed normally.
In multi- choice programming environment, we formulate a multi- choice and multi-objective model for solving an integrated production planning problems. Computations of the multi- choice model has been performed with the real production data to find the efficiency of the methodology

Keywords : Operational research , Optimal Solution , Linear Programming , Multi-choice

Content

Chapter 1

Literature Review

By the beginning of 1950's, operations research is considered the most familiar method of summarizing any real life problem in order to look for an acceptable and reasonable solution called optimal solution. Operational research deals with the application of advanced analytically methods to help make better decisions.

Most life phenomena, such as chemical reaction, industrial accounting systems, and motion laws are efficiently expressed by mathematical equations.

In the field of optimization, we try to convert a real life problem to a symbolic form known as mathematical model which gives an efficient and clear explanation to the problem of interest, also we solve the mathematical model in order to determine an optimal solution for that real life problem.

The main goal of any optimization problem is to minimize or maximize a mathematical expression. These mathematical expression which called objective function (sometimes there are more than one objective function) may be a linear combination of the basic variables defined in the problem and sometimes non-linear. In addition, we have some restrictions known as constraints which are imposed on the problem. These constraints expressed as mathematical inequalities or equations. Moreover, additional restrictions imposed on the basic variables of the problem.

The logical approach to solve a decision making problem is to formulate the information of the problem to a mathematical model. There is no a unique technique to solve all mathematical models that arise through treating different optimization problems. In other words, our solution of the mathematical model mainly depend on the complexity and the nature of the problem. The most famous mathematical model is the linear programming (LP) model, where the left side of the objective function and the constraints are all linear functions in the basic variables. These type of models are usually constructed in production problem where we wish to minimize the cost or maximize the profit. By the solution of such problems,

we look for the "best" value obtainable under those conditions. This value called "best" value because it is not only satisfies the constraint but also achieves our desired aim to maximize minimize the objective function of interest. In other words, the solution of the optimization problem not only satisfying all constraints but also achieves the best minimum or maximum value of the mathematical expression (objective function).

Any optimization problem involving n basic variables and m constraints can be formulated by the following well known mathematical model:

Find $Y = (y_1, y_2, y_3, \ldots., y_n)$ so as to:

$$Max\ (Min) : F = \sum_{r=1}^{n} b_r\ y_r \tag{1.1}$$

subject to:

$$\sum_{r=1}^{n} a_{ir}\ y_r \leq c_i \quad , i = 1, 2, \ldots., m \tag{1.2}$$

$$y_r \geq 0 \quad , r = 1, 2, \ldots., n \tag{1.3}$$

Linear programming plays an important role in optimization techniques, which considered one of the most significant areas in mathematics. This field of study is extensively used in real life problem to organize and allocate resources. This type of problems involves a hundreds of variables or more.

However, any mathematical model describing an optimization problem contains a large number of parameters whose values are given by the decision makers. In the standard approach, it is required to set just one value to each parameter which is mentioned before. Nevertheless , neither the decision-maker nor experts know the value of those parameters precisely. In most cases, they mainly depend on noisy data comes from old statistical inference. In fact, the values given by the decision makers or experts for the parameters of the optimization problem are not accurate and doubtful. So, there are many approaches and points of view to handle this type of problems of decision making under noisy data. The best approaches to handle an uncertainty optimization problem of that type used stochastic and fuzzy programming. However, in this case, the parameters attached with the optimization problems are multi- choice in nature. This type of optimization problems known as the multi- choice (LP) problems.

The reasonable approach for solving the multi- choice problem depends mainly on converting the right side of each constrain which contains more than one value to a mathematical expression by introducing a set of binary variables. This process could be done in different tech-

niques. Somehow, for each possible choice for the values of the binary variables we obtain exactly one value for the mathematical expression in the right side of each constraint. Consequently, this enable us to treat our problem of interest as an ordinary optimization problem. The methodology procedure of the solution depend on constructing a mathematical expression involving a finite number of binary variables and the multi- choice values to obtain a linear or non-linear mathematical expression. However, number of constraints will increase by imposing extra restrictions on the used binary variables to guarantee that the mathematical expression in the right side of the constraint gives just one value. In most cases, the resulting mathematical expression is non-linear, and the multi- choice problem in this case treated as a mixed-integer programming problem.

In some situations, for some loss of experience or other reasons, the management of the organization (decision makers) are unable to assign an accurate value for the parameters which represent the source limits. So, they mention a multiple number of choices for those parameters. In fact, the straight forward thinking to handle this type of optimization problem is difficult and computationally expensive since the problem will turned into a

huge number of ordinary (LP) problems. The researcher classified this kind of problems as multi- choice Programming problems. To solve this type of problems, there is no direct method to select an appropriate value from those multi- choice values of each parameter. Therefore, to develop a methodology that give the optimal solution for such problems is considered one of the most challenging points in this research area.

The dissertation highlights the formulation of multi-choice problem considering the resource limits as multi-choice type parameters. It covers a detailed description of transformation techniques in order to handle the multi-choice (LP) problem.

In this dissertation, we contribute to this research area by presenting a new technique to convert the multi- choice problem to an usual (LP) problem. The converting techniques due to the results of some other papers transforms the multi- choice problem to a non-linear programming problem. In both cases the problem turned to a mixed integer linear or non-linear programming problem, which can be treated easily using the specialized software as Lingo or Matlab [48,49].

In 1964, Healey introduced the notion of multi- choice problem, where more than one possible value as an alter-