بسم الله الرحمن الرحيم

شبكة المعلومـــــات الجامعية

التوثيق الالكتروني والميكروفيلم

شبكة المعلومات الجامعية

جامعة عين شمس

التوثيق الالكتروني والميكروفيلم

﴿ قـــسـم ﴾

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لم ترد بالأصـــل

Zagazig University
Benha Branch - Faculty of Science

ON FUZZY TOPOLOGICAL SPACES

Thesis

Submitted By

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CONTENTS

PREFACE

PREFACE

In his classic paper [69] of 1965, Zadeh introduced the notion of fuzzy sets and their operations. Subsequently, Chang [12], Wong [66], Lowen [35] and others applied some basic concepts from general topology to fuzzy sets and developed a theory of fuzzy topological spaces. We denote by FTS the category of Lowen's fuzzy topological spaces and thier continuous functions.

The thesis consists of four chapters :
The first chapter in this thesis contains basic ideas on fuzzy sets and fuzzy topological spaces, together with their basic properties. The second, third and fourth chapters consist of a new research work done by us. In the second chapter, we introduce for each continuous triangular norm T, a new structure that is a fuzzy T-proximity space, those generalize the fuzzy proximity spaces given in [7] by G. Artico and R. Moresco, which now coincide with our fuzzy T-proximity spaces when T = Min. Some properties of this notion are given. We generate a fuzzy T-proximity given by a classical proximity. Moreover, we introduce a notion of fuzzy T-proximal neighbourhood systems, which are in one-to-one correspondence with the fuzzy T-proximity spaces. Also, we show that every Höhle fuzzy T-uniformity [21] induces a fuzzy T-proximity and that the fuzzy topologies generated by the two structures coincide. Also, we characterize a fuzzy T-proximity, uniquely in terms of its behaviour

on crisp fuzzy subsets.

In the third chapter we study for each lower semicontinuous triangular norm T, a new type of Lowen fuzzy topological spaces which we call the fuzzy T-neighbourhood spaces. Those generalize the fuzzy neighbourhood spaces introduced by R. Lowen in [40] (which now correspond to the case T = Min). Also, we establish chracterizations and aspects of good behaviour for those spaces, including that there is a one-to-one correspondence between fuzzy T-neighbourhood spaces and fuzzy T-neighbourhood systems, we briefly study their level topologies, and we show that all topologically generated spaces are fuzzy T-neighbourhood spaces. We prove that the category T-FNS, of fuzzy T-neighbourhood spaces and continuous functions between them, is a topological category. We show that it is closed in FTS under the formation of both optimal lifts of sources and co-optimal lifts of sinks. Also, we define a functor from the category of fuzzy T-uniform spaces into the category T-FNS of fuzzy T-neighbourhood spaces. We study the continuity of functions in the category T-FNS in terms of fuzzy T-neighbourhood systems.

In the fourth chapter, we introduce the new concept of fuzzy T-neighbourhood systems of fuzzy subsets. This helps us to introduce and study a number of separation properties of fuzzy topological spaces and fuzzy T-neighbourhood spaces, all of which are extensions of the classical R_1 property, regularity and normality in topological spaces. They also generalize existing axioms for the case T = Min, i.e. for fuzzy neighbourhood spaces [9, 29, 44, 51,

52, 55]. Also, we show that for each triangular norm T, the axioms T-R_1 and T-regularity are initial in FTS. We study some relations among these separation axioms, we show that all these separation axioms are good extensions in the sense of R. Lowen [37]. Also, we introduce an axiom of T-complete regularity, on fuzzy topological spaces. We show that it is equivalent to fuzzy T-uniformizability.

CHAPTER (I)

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BASIC IDEAS IN FUZZY TOPOLOGICAL SPACES

Introduction

In this chapter, we introduce definitions and main results of fuzzy subsets, triangular norms, and fuzzy topological spaces, which are required for our study in this thesis.

We group them into three sections.

The first section contains the ideas of fuzzy subsets and their α-cuts, the concepts of prefilters, prefilterbases and triangular norms.

In the second section we deal with the concepts of fuzzy topological spaces, and in the third section the α-level topologies of a fuzzy topological space are investigated.

I.1 Preliminaries

We use the customary notations $I = [0,1]$, $I_0 =]0,1]$, $I_1 = [0,1[$ and $I_{0,1} =]0,1[$ for unit intervals of real numbers. The notion of a fuzzy subset μ of a universe X (written $\mu \in I^X$) was introduced by Zadeh in his classical paper [69]. This is a function from X to I. The real number $\mu(x)$ for $x \in X$, is called the membership value of x in μ. It is the degree to which x possesses the property attributed to the fuzzy subset μ. We denote the constant fuzzy subset of X with value $\alpha \in I$ by the symbol $\underline{\alpha}$ $(\in I^X)$. We denote the fuzzy