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A Study of the effect of the electromagnetic field on non-Newtonian fluids flow with heat and mass transfer and its applications

THESIS
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SUMMARY

Most of the natural phenomena can be transferred to a system of partial differential equations that control these phenomena with appropriate boundary and initial conditions. By solving these eigen-value problems either by analytical or numerical methods to find usually the velocity, the temperature and the concentration in fluid mechanics flow. We can study the effects of the different affecting parameters on these phenomena.

In this thesis, we interest of magnetohydrodynamis of non-Newtonian fluids with heat and mass transfer in the presence of external magnetic fields through different geometric shapes.

This thesis have many important physical applications in a variety of branches of science such as industrial, chemical, biological, space and medical science.

This thesis consists of six chapters, appendix and a list of references related to the subject of the thesis. This chapters can be summarized as follows:

Chapter 1:

This chapter gives an introduction of fluid mechanics, It includes the items which are closely related to the subject of the thesis such as:

1. Fluid mechanics
2. Some properties of fluids
3. Classification of fluids
4. Classification of non-Newtonian fluids
5. Basic equations
6. Flow through porous media
7. Heat transfer
8. Mass transfer

9. Magnetohydrodynamics (MHD) flow
10. Harmonic motion
11. Pulsatile flow
12. Perturbation method
13. Some previous studies

Chapter (2)

Through this chapter, we studied the influence of varying magnetic field on the unsteady magnetohydrodynamics oscillatory flow of an incompressible Kuvshinsk's fluid through porous medium between two permeable rotating parallel plates. The analytical formulae of the velocity and the magnetic field components have been obtained and the effects of different parameters such as the magnetic parameter, the rotating parameter, the permeability coefficient, the Reynolds number, the magnetic Reynolds number and the elasticity parameter have been studied and illustrated graphically through a set of figures.

(This was sent to be published in the Calcutta Mathematical Journal).

Chapter (3)

The aim of this chapter is to study the effect of an Ohmic dissipation on the pulsatile flow of an incompressible non-Newtonian fluid obeying Burgers' model with heat transfer. The fluid is flowing in a circular pipe through a porous medium in the presence of a uniform magnetic field acting parallel to the radius of the pipe. The analytical solutions of the momentum and heat equations have been obtained in terms of the modified Bessel function. Finally, we studied the effects of various parameters such as: the magnetic parameter and the permeability coefficient on the velocity and temperature profiles form.

(This problem was published in Special Topics & Reviews in Porous Medium – An International Journal Vol. 1, No. 3, PP. 243-255, 2011).

Chapter (4)

In this chapter, we studied the hydromagnetic Couette flow of an electrically conducting and incompressible Burgers' fluid with heat and mass transfer. The motion is confirmed between two infinite non-conducting parallel plates through a porous medium under a uniform transverse magnetic field as a rotating system. The lower plate is at rest, while the upper one is oscillating in its own plane. The distributions of the velocity, temperature and concentration are derived by using Lightill's perturbation technique. The effects of the various physical parameters of the problem on the velocity, temperature and concentration formulas are shown graphically and discussed.

(This problem was sent to be published in Journal of Porous Medium).

Chapter (5)

Through this chapter, we discussed the problem of unsteady flow with heat and mass transfer of electrically conducting non-Newtonian fluid of Burgers' model through porous medium in a vertical channel. The system is stressed by a varying magnetic field and under the actions of diffusion-thermo, heat absorption and chemical reaction. The analytical formulas of the velocity, temperature and concentration have been obtained using a perturbation technique. The effects different parameters such as the magnetic parameter M, the permeability parameter k, the Schmidt number S_c, the Prandtl number P_r, the Grashof number G, the modified Grashof number G_r and the reaction rate δ on these formulae have been discussed and illustrated graphically.

(This problem was sent to be published in Chemical Engineering Communications Journal).

Chapter (6)

In this chapter, we obtained an analytical solution of the M H D flow of an electrically conducting and incompressible non- Newtonian fluid obeying the Oldroyd-B model in a porous medium between two vertical circular cylinders is investigated. The motion is considered under the effects of couple stresses and a uniform external magnetic field. The outer cylinder is at rest while the inner one is rotating with a uniform velocity. The perturbation technique under the small Reynolds number R_e is considered to solve the momentum, heat and concentration equations. The formulae of the velocity, temperature and concentration distributions are obtained in terms of the Bessel function and Euler Gamma constant. The graphs of the velocity, temperature and concentration are plotted and illustrated for different values of the non-dimensional parameters of the problem.

(This problem was sent to be published in International Journal of Applied Mathematics and Physics).

Chapter (1)

General Introduction

(1) Fluid mechanics

Fluid mechanics is one of the most important parts of the recent activities concerning engineering and technological developments. Fluid mechanics is that branch of science, which is concerned to the study of the motion of fluids or that of bodies in contact with fluids.

Fluid mechanics is a sub discipline of continuum mechanics, as illustrated in the following table:

<table>
<tr><td rowspan="4">Continuum mechanics: the study of the physics of continuous materials</td><td rowspan="2">Solid mechanics: the study of the physics of continuous materials with a defined rest shape.</td><td colspan="2">Elasticity: which describes materials that return to their rest shape after an applied stress.</td></tr>
<tr><td>Plasticity: which describes materials that permanently deform after a large enough applied stress.</td><td rowspan="2">Rheology: the study of materials with both solid and fluid characteristics</td></tr>
<tr><td rowspan="2">Fluid mechanics: the study of the physics of continuous materials which take the shape of their container.</td><td>Non-Newtonian fluids</td></tr>
<tr><td colspan="2">Newtonian fluids</td></tr>
</table>

Table (1): Relationship between fluid mechanics and continuum mechanics.

(1.1) Definition of a fluid

A fluid is a substance which cannot resist a shear force or stress without moving as can a solid. Fluids are usually classified as liquids or gases. A liquid has intermolecular the other hand, consists of molecules in motion which collide with each other tending to disperse it so that a gas has no set volume or shape [1].

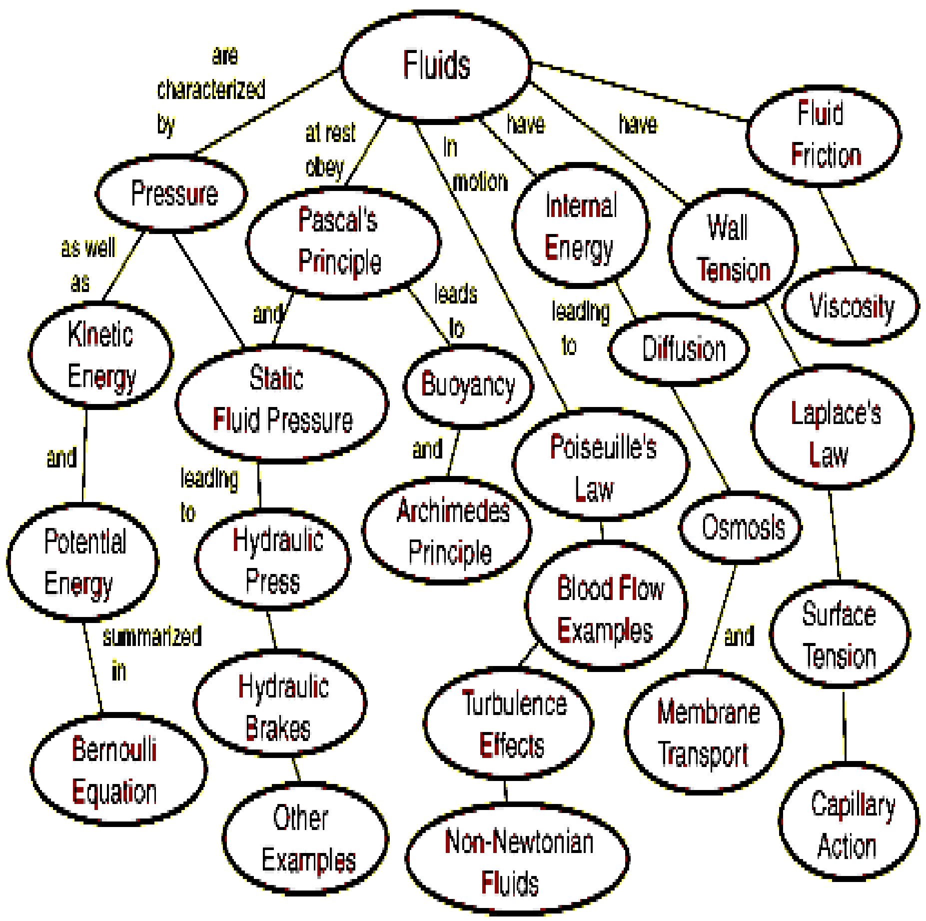

Figure (1)

Many applications of fluid mechanics make it one of the most vital and fundamental of

all engineering and applied scientific studies. The flow of fluids in pipes and channels makes fluid mechanics of importance in civil engineering. The study of fluid machinery such as pumps, compressors, heat exchangers, jet and rocket engines makes fluid mechanics of importance to mechanical engineers. And today in modern engineering, many new disciplines combine fluid mechanics with classical disciplines. For example, fluid mechanics and electromagnetic theory are studied together as magnetohydrodynamic (MHD).

As good biological examples on fluids there are in the human body, approximately 56 percent of the human body consist of fluids. These fluids are composed largely of water. Therefore , water is the major component of living substances. Regarding the human body, we speak of fluid compartments or spaces. These are intracellular fluid, the interstitial fluid, and the circulating (plasma) fluid, see figure (2).

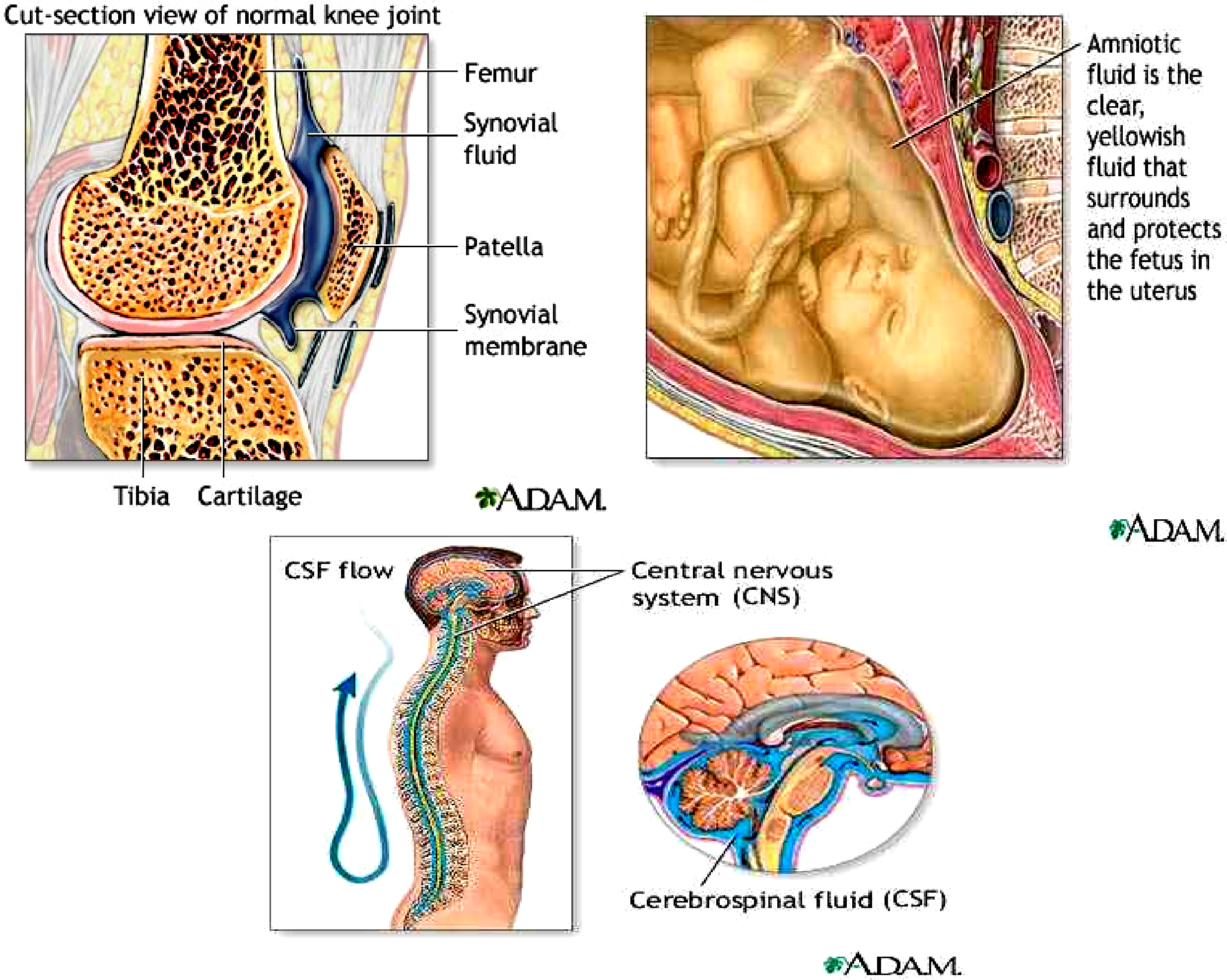

Figure (2):biological examples on fluids in the human body.

(2) Some properties of fluid

(2.1) Pressure

The pressure in a static fluid is defined as the normal compressive force per unit area acting on a surface immersed in the fluid. The pressure at a point is isotropic in a fluid at rest, such as an isotropic pressure is called hydrostatic pressure. This is the pressure used in thermodynamics (gas law). If the pressure varies from place to place in the fluid, a net pressure force would exist on any fixed volume of fluid It must be balanced by a body force such as gravity, or else the fluid will move, the pressure force generating acceleration in the fluid.

(2.2) Stress and strain

Stress is a measure of the internal reaction between elementary particles of a material in resisting separation, compaction, or sliding that tend to be induced by external forces. Total internal resisting forces are resultants of continuously distributed normal and parallel forces that are of varying magnitude and direction and are acting on elementary areas throughout the material. These forces may be distributed uniformly or non-uniformly. Stresses are identified as tensile, compressive, or shearing, according to the straining action. Strain is a measure of deformation [2] such as:

(I) Linear strain, the change of length per unit of linear dimensions.

(II) Shear strain, the angular rotation in radians of an element undergoing change of shape by shearing forces.

(III) Volumetric strain, the change of volume per unit of volume. The strains associated with stress are characteristic of the material.

Strains, completely recoverable on removal of stress, are called elastic strains. Above a critical stress, both elastic and plastic strains exist, and that part remaining after unloading represents plastic deformation called inelastic strain. Inelastic strain reflects internal changes in the crystalline structure of the metal. Increase of resistance to continued plastic deformation due to more favourable rearrangement of the atomic structure is strain hardening.

A stress-strain diagram is a graphical representation of simultaneous values of stress and strain observed in tests and indicates material properties associated with both elastic and inelastic behaviour, see figure (3). It indicates significant values of stress-accompanying changes produced in the internal structure.

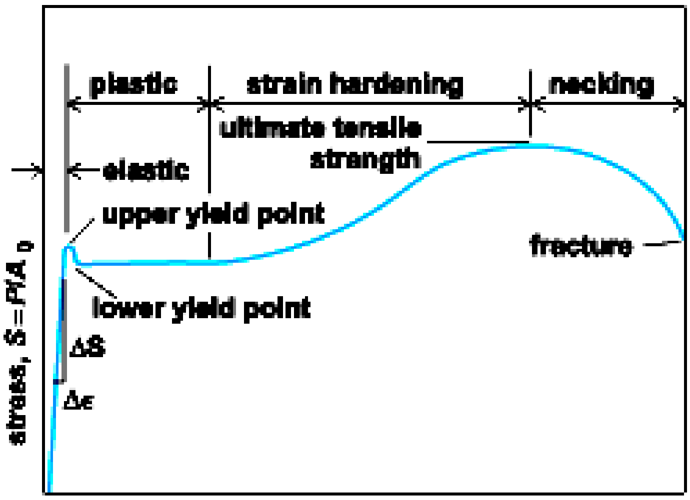

Figure (3): Stress-strain diagram for a low-carbon steel,

where ΔS = change in stress, $\Delta \varepsilon$ = change in strain, P = force and A_0 = area of cross section.

Shear strain is a strain that acts parallel to the surface of a material that it is acting on. Normal strain acts perpendicular to the face of that it is acting on. There are two ways to interpret shear strain: the average shear strain and the engineering shear strain.

(2.3) Viscosity

All fluids have viscosity in varying degrees which causes friction. If the friction is negligible, we say that the fluid flow is ideal. The coefficient of viscosity of a fluid may be defined as the tangential force required per unit area (shearing stress) to a unit velocity gradient. The dimensions of the coefficient of viscosity μ, see figure (4), can be defined as:

$$\mu = \frac{Force/Area}{Velocity/height}. \tag{1}$$

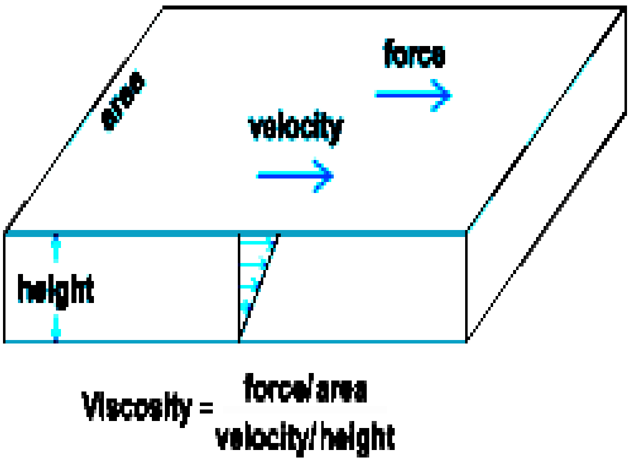

Figure (4): illustrates steady shear flow of a fluid between a fixed plate and a parallel plate, illustrating the concept of viscosity.

(2.4) kinematic viscosity

Kinematic viscosity is the ratio between the coefficient of viscosity μ and the mass density ρ (the mass of unit volume of the fluid, which has the dimensions ML^{-3}).

The dimensions of the kinematic viscosity ν can be defined as:

$$\nu = \frac{\mu}{\rho}. \tag{2}$$

(2.5) Thermal conductivity

The coefficient of thermal conductivity can be defined from Fourier's law of heat conduction, which supposed that two parallel layers of fluid at a distance d a part, are kept at different temperatures T_1 and T_2. Fourier noticed that a flow of heat is set up through the layer such that the quantity of heat $Q(T)$ transferred through unit area in a unit time. It is directly proportional to the difference of temperature between the two layers and inversely proportional to the distance d [3], thus he found

$$Q(T) = \frac{\alpha(T_1 - T_2)}{d}, \quad (3)$$

where α is the constant of proportionality. It is known as the coefficient of thermal conductivity.

If the distance d between two layers of fluid is infinitesimal the above law can be written in the differential form as:

$$Q(T) = \alpha \frac{dT}{dy}, \quad (4)$$

which is the Fourier's law of heat conduction.

(3) Classification of fluids

Depending on the relationship between shear stress, and rate of strain and its derivatives, fluids can be characterized as figure (5)

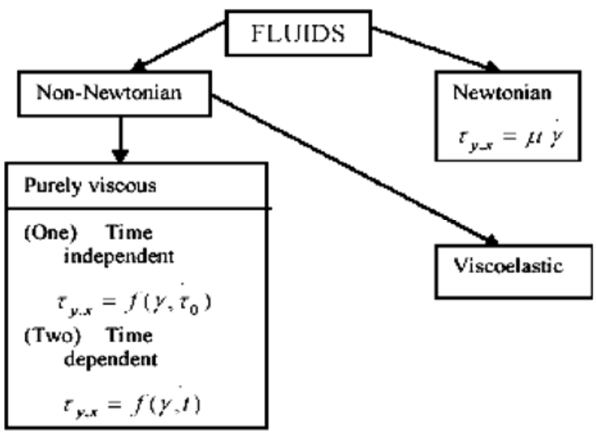

Figure (5): illustrates the characteristics of fluids.