Cairo University

# NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Aerospace Engineering

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT
2015

# NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Aerospace Engineering

Under the Supervision of

Prof.Dr. Mohamed M. Abdelrahman

........................

Professor of Aerodynamics

Aerospace Engineering Department

Faculty of Engineering, Cairo University

Prof.Dr. Farouk M. Owis

.......................

Professor of Propulsion and Jet Engines

Aerospace Engineering Department

Faculty of Engineering, Cairo University

Dr. Hesham M. Elbanna

.........................

Assistant Professor

Aerospace Engineering Department

Faculty of Engineering, Cairo University

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT
2015

# NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the
Faculty of Engineering at Cairo University
in Partial Fulfillment of the
Requirements for the Degree of
MASTER OF SCIENCE
in
Aerospace Engineering

Approved by the Examining Committee:

---

Prof.Dr. Mohamed M. Abdelrahman, Thesis Main Advisor

---

Prof.Dr. Farouk M. Owis, Member

---

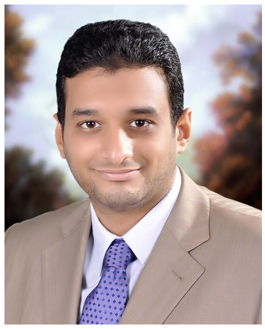
Prof.Dr. Atef O. Sherif, Internal Examiner

---

Prof.Dr. Ali M. Al-Bahi, External Examiner
Aeronautical Engineering, Faculty of Engineering, King Abdulaziz University,
Kingdom of Saudi Aarabia

FACULTY OF ENGINEERING, CAIRO UNIVERSITY
GIZA, EGYPT
2015

**Engineer's Name:** Mohammad Ahmad Mohammad Ahmed Alhawwary
**Date of Birth:** 02/05/1987
**Nationality:** Egyptian
**E-mail:** mhawwary@eng.cu.edu.eg
**Phone:** 01002963004
**Address:** Fariskour, Damietta, 34611


**Registration Date:** 1/10/2010
**Awarding Date:** .../.../2015
**Degree:** Master of Science
**Department:** Aerospace Engineering

**Supervisors:**

Prof.Dr. Mohamed M. Abdelrahman
Prof.Dr. Farouk M. Owis
Dr. Hesham M. Elbanna

**Examiners:**

Prof.Dr. Ali M. Al-Bahi (External examiner)
Aeronautical Engineering, Faculty of Engineering,
King Abdulaziz University, Kingdom of Saudi Aarabia
Prof.Dr. Atef O. Sherif (Internal examiner)
Prof.Dr. Mohamed M. Abdelrahman (Thesis main advisor)
Prof.Dr. Farouk M. Owis (Member)


**Title of Thesis:**

Numerical Solution of the Navier Stokes Equations using the Higher Order Spectral Difference Method


**Key Words:**

Navier Stokes equations; Higher Order methods; Spectral Difference; Unstructured Quadrilaterals; Laminar Compressible flows; Airfoil Spoiler Aerodynamics



**Summary:**

Higher order discretization on unstructured grids can be a possible avenue for improving the predictive capabilities of numerical flow simulations. Therefore, a two dimensional Navier Stokes flow solver on unstructured quadrilateral grids has been developed, associated with an elliptic structured grid generation module. This solver is based on the higher order Spectral Difference method for spatial discretization. The solver is tested for validity with the published data and the expected order of accuracy of the method is also achieved.

# Acknowledgements

All gratitude is due to the Almighty Allah.

I would like to thank my main supervisor, Prof. Dr. Mohamed Madbuli Abdelrahman, for his continuous support, valuable discussions and guidance during the course of this work. I am also grateful to him for the knowledge and education he gave to me in my undergraduate studies which have a great influence on my academic personality. Also, I would like to thank my co-advisor, Prof. Dr. Farouk Owis, for his great help in this work, his suggestions for the thesis topic and the fruitful discussions we had in finite volume basics. Dr. Hesham Elbanna, had contributed significantly in my understanding of classical and unsteady aerodynamics in my undergraduate studies which is reflected in my thesis work. In addition, he has a kind personality and is always welling to help his students without hesitations with endless support. I wish also to thank my examination committee Prof. Dr. Ali Al-Bahi and Prof. Dr. Atef Sherif for their valuable comments and discussions on my thesis content which for sure resulted in better final works.

Additionally, I wish to thank all who had helped me through all the stages of this work in various ways. Particularly, I would like to express my deep gratitude to Dr. Mohammad Baher Azab for his tremendous support and technical discussions that helped me so much in debugging the developed solver and overcoming many difficulties, especially in the early stages of this work. I am so grateful to Dr. Mohammad El-Beltagy, for providing me with his grid connectivity module, the time he dedicated for me, and continuous encouragement. Also, I am so grateful to Associate Prof. Dr. Lamyaa El-Gabry, Dr. Ahmed Altaweel, and Dr. Amr Guaily for their help and support during my graduate studies and research. Special thanks go to Dr. Haithem Ezzat Taha for the encouragement and confidence he gave to me, which helped me so much in publishing my first scientific paper.

Furthermore, I would like to acknowledge all the faculty and staff members of the Aerospace Engineering Department at Cairo University for providing me with an outstanding engineering experience and their engraved teachings. Particularly, I wish to express my sincere appreciations to Prof. Dr. Ali Hashem, Prof. Dr. Atef Sherif, Prof. Dr. Gamal El-Bayoumi, Prof. Dr. Hani Negm, Prof. Dr. Edward Sadek, and Prof. Dr. Mohamed Khalil. In addition, I would like to express my special gratefulness to Associate Prof. Dr. Basman Elhadidi for the great knowledge he gave to me in fluid mechanics through my undergraduate and postgraduate studies. He had always provided me with continuous advices and encouragement that helped in building my academic knowledge.

My parents have always been my first mentors, and my strongest supports through all the obstacles and difficulties I have encountered in my life. I wish to express my special appreciation to my mother for the effort that she had exerted in my early education phases and for her prayers. Also, my father has never hesitated to provide me with the advice and support whenever I needed them. I can not forget my brothers too; Ahmed, Zeyad, Omran, and Gihana for their help and support. Finally, my kind acknowledgment is for my wife, Hadeer, who had been my weal and woe partner through almost tow years of my academic and research work, supporting and encouraging me, and making my home an earth paradise.

# Table of Contents

# List of Tables

# List of Figures

# Nomenclature

| | |
|---|---|
| $[A_n]$ | Normal Jacobian matrix |
| $\alpha$ | Angle of attack |
| $\varphi(\vec{\xi})$ | Shape functions for the isoparametric transformation |
| $\epsilon_s$ | Entropy error |
| $\gamma$ | Specific heat ratio, 1.4 for air |
| $\mu$ | Dynamic viscosity coefficient |
| $\phi^{fp}$ | Lagrange interpolation polynomial for the flux points |
| $\Phi_l^{fp}(\vec{\xi})$ | Flux basis polynomials |
| $\psi^{sp}$ | Lagrange interpolation polynomial for the solution points |
| $\Psi_i^{sp}(\vec{\xi})$ | Solution basis polynomials |
| $\rho$ | Density |
| $\tau_{xx}, \tau_{xy}, \tau_{yy}$ | Shear stress tensor elements |
| $\tilde{\vec{F}}$ | Total flux vector in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{\vec{F}}_c$ | Convective flux vector in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{\vec{F}}_d$ | Diffusive flux vector in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{f}_c, \tilde{g}_c$ | $\xi, \eta-$components of convective flux vector, respectively, in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{f}_c^{common}, \tilde{g}_c^{common}$ | Interface common convective flux components in computational domain |
| $\tilde{f}_d, \tilde{g}_d$ | $\xi, \eta-$components of diffusive flux vector, respectively, in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{f}, \tilde{g}$ | $\xi, \eta$-components of the total flux vector in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{f}^k(\vec{\xi}), \tilde{g}^k(\vec{\xi})$ | Computational flux polynomials in cell $k$ |
| $\tilde{Q}$ | conserved variables in cell local computational coordinate $\vec{\xi}$ |
| $\tilde{Q}_f^k, \tilde{Q}_g^k$ | Computational conserved variables polynomials defined at $\xi, \eta$-flux points, respectively for cell $k$ |
| $\vec{\nabla}$ | Gradient operator vector in the Cartesian coordinates system: $[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]^T$ |

| | |
|---|---|
| $\vec{\nabla}_{\vec{\xi}}$ | Gradient operator vector in the computational coordinates system: $[\frac{\partial}{\partial\xi}, \frac{\partial}{\partial\eta}]^T$ |
| $\vec{\xi}$ | Vector of computational coordinates system |
| $\vec{F}$ | Total flux vector in physical domain |
| $\vec{F}_c$ | Convective flux vector in physical domain |
| $\vec{F}_d$ | Diffusive flux vector in physical domain |
| $\vec{n}$ | Unit normal vector in the physical Cartesian coordinates system |
| $\vec{x}$ | Vector of Cartesian coordinates system |
| $\vec{\nabla}Q^k \approx \vec{\Theta}(\vec{\xi})$ | Conserved variables gradient polynomial |
| $\widetilde{Res_i}(\vec{\xi}^{sp})$ | Residual of the governing equations in computational domain |
| $\xi, \eta$ | Computational coordinates |
| $\xi^{fp}$ | Flux points coordinates in 1D |
| $\xi^{sp}$ | Solution points coordinates in 1D |
| $\xi_x, \xi_y, \eta_x, \eta_y$ | Metrics of the transformation |
| $c$ | Speed of sound |
| $C_D$ | Drag coefficient |
| $C_L$ | Lift coefficient |
| $Cp$ | Pressure Coefficient |
| $E_{c,n}$ | Interface normal convective flux component in physical Cartesian coordinate system |
| $e_t$ | Specific total energy |
| $f$ | x-component of the total flux vector in physical Cartesian coordinates |
| $f_c, f_d$ | x-components of the convective and diffusive flux vectors, respectively, in physical Cartesian coordinates |
| $f_r$ | Frequency in hertz |
| $g$ | y-component of the total flux vector in physical Cartesian coordinates |
| $g_c, g_d$ | y-components of the convective and diffusive flux vectors, respectively, in physical Cartesian coordinates |
| $J$ | Jacobian of the transformation |
| $L_c$ | Characteristic length |

| | |
|---|---|
| $M_\infty$ | Free stream Mach number |
| $N$ | Number of solution point in 1D cells |
| $N^{fp}$ | Number of flux points in d-dimensional elements |
| $N^{sp}$ | Number of solution points in d-dimensional elements |
| $n_x, n_y$ | Components of the physical unit normal vector $\vec{n}$ |
| $Nn$ | Total number of geometric nodes/vertices used to define the physical element |
| $P$ | Pressure |
| $p$ | Polynomial order |
| $P_t$ | Total pressure |
| $Pr$ | Prandtl number, 0.72 for air |
| $Q$ | Conserved variables in physical domain |
| $Q^k(\vec{\xi}), \tilde{Q}^k(\vec{\xi})$ | Physical and computational solution polynomials in cell $k$, respectively |
| $q_x, q_y$ | Heat flux vector components in x,y directions, respectively |
| $R_x, R_y$ | Total forces in x,y-directions, respectively |
| $Re_v$ | Reynolds number |
| $Rn$ | One-dimensional Riemann invariants |
| $s$ | Entropy |
| $S_A$ | Area of the nozzle for the quasi-one dimensional nozzle flow |
| $S_t$ | Strouhl number |
| $T$ | Temperature |
| $t$ | Time |
| $T_t$ | Total temperature |
| $u, v$ | Velocity components in x,y directions, respectively, $m/s$ |
| $V_\infty$ | Free stream velocity |
| $V_n$ | Normal velocity component in the physical Cartesian coordinate system |
| $V_t$ | Tangent velocity component in the physical Cartesian coordinate system |
| $x, y$ | Cartesian coordinates |