



# **NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD**

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Aerospace Engineering

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2015

# **NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD**

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Aerospace Engineering

Under the Supervision of

Prof.Dr. Mohamed M. Abdelrahman

Prof.Dr. Farouk M. Owis

.....

.....

Professor of Aerodynamics

Professor of Propulsion and Jet Engines

Aerospace Engineering Department

Aerospace Engineering Department

Faculty of Engineering, Cairo University

Faculty of Engineering, Cairo University

Dr. Hesham M. Elbanna

.....

Assistant Professor

Aerospace Engineering Department

Faculty of Engineering, Cairo University

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2015

# **NUMERICAL SOLUTION OF THE NAVIER STOKES EQUATIONS USING THE HIGHER ORDER SPECTRAL DIFFERENCE METHOD**

By

Mohammad Ahmad Mohammad Ahmed Alhawwary

A Thesis Submitted to the  
Faculty of Engineering at Cairo University  
in Partial Fulfillment of the  
Requirements for the Degree of  
**MASTER OF SCIENCE**  
in  
Aerospace Engineering

Approved by the Examining Committee:

---

Prof.Dr. Mohamed M. Abdelrahman, Thesis Main Advisor

---

Prof.Dr. Farouk M. Owis, Member

---

Prof.Dr. Atef O. Sherif, Internal Examiner

---

Prof.Dr. Ali M. Al-Bahi, External Examiner

Aeronautical Engineering, Faculty of Engineering, King Abdulaziz University,  
Kingdom of Saudi Arabia

FACULTY OF ENGINEERING, CAIRO UNIVERSITY  
GIZA, EGYPT  
2015

**Engineer's Name:** Mohammad Ahmad Mohammad Ahmed Alhawwary  
**Date of Birth:** 02/05/1987  
**Nationality:** Egyptian  
**E-mail:** mhawwary@eng.cu.edu.eg  
**Phone:** 01002963004  
**Address:** Fariskour, Damietta, 34611  
**Registration Date:** 1/10/2010  
**Awarding Date:** .../.../2015  
**Degree:** Master of Science  
**Department:** Aerospace Engineering



**Supervisors:**

Prof.Dr. Mohamed M. Abdelrahman  
Prof.Dr. Farouk M. Owis  
Dr. Hesham M. Elbanna

**Examiners:**

Prof.Dr. Ali M. Al-Bahi	(External examiner)
Aeronautical Engineering, Faculty of Engineering, King Abdulaziz University, Kingdom of Saudi Arabia	
Prof.Dr. Atef O. Sherif	(Internal examiner)
Prof.Dr. Mohamed M. Abdelrahman	(Thesis main advisor)
Prof.Dr. Farouk M. Owis	(Member)

**Title of Thesis:**

Numerical Solution of the Navier Stokes Equations using the Higher Order  
Spectral Difference Method

**Key Words:**

Navier Stokes equations; Higher Order methods; Spectral Difference; Unstructured Quadrilaterals; Laminar Compressible flows; Airfoil Spoiler Aerodynamics

**Summary:**

Higher order discretization on unstructured grids can be a possible avenue for improving the predictive capabilities of numerical flow simulations. Therefore, a two dimensional Navier Stokes flow solver on unstructured quadrilateral grids has been developed, associated with an elliptic structured grid generation module. This solver is based on the higher order Spectral Difference method for spatial discretization. The solver is tested for validity with the published data and the expected order of accuracy of the method is also achieved.

# Acknowledgements

All gratitude is due to the Almighty Allah.

I would like to thank my main supervisor, Prof. Dr. Mohamed Madbuli Abdelrahman, for his continuous support, valuable discussions and guidance during the course of this work. I am also grateful to him for the knowledge and education he gave to me in my undergraduate studies which have a great influence on my academic personality. Also, I would like to thank my co-advisor, Prof. Dr. Farouk Owis, for his great help in this work, his suggestions for the thesis topic and the fruitful discussions we had in finite volume basics. Dr. Hesham Elbanna, had contributed significantly in my understanding of classical and unsteady aerodynamics in my undergraduate studies which is reflected in my thesis work. In addition, he has a kind personality and is always willing to help his students without hesitations with endless support. I wish also to thank my examination committee Prof. Dr. Ali Al-Bahi and Prof. Dr. Atef Sherif for their valuable comments and discussions on my thesis content which for sure resulted in better final works.

Additionally, I wish to thank all who had helped me through all the stages of this work in various ways. Particularly, I would like to express my deep gratitude to Dr. Mohammad Baher Azab for his tremendous support and technical discussions that helped me so much in debugging the developed solver and overcoming many difficulties, especially in the early stages of this work. I am so grateful to Dr. Mohammad El-Beltagy, for providing me with his grid connectivity module, the time he dedicated for me, and continuous encouragement. Also, I am so grateful to Associate Prof. Dr. Lamyaa El-Gabry, Dr. Ahmed Altaweel, and Dr. Amr Guaily for their help and support during my graduate studies and research. Special thanks go to Dr. Haithem Ezzat Taha for the encouragement and confidence he gave to me, which helped me so much in publishing my first scientific paper.

Furthermore, I would like to acknowledge all the faculty and staff members of the Aerospace Engineering Department at Cairo University for providing me with an outstanding engineering experience and their engraved teachings. Particularly, I wish to express my sincere appreciations to Prof. Dr. Ali Hashem, Prof. Dr. Atef Sherif, Prof. Dr. Gamal El-Bayoumi, Prof. Dr. Hani Negm, Prof. Dr. Edward Sadek, and Prof. Dr. Mohamed Khalil. In addition, I would like to express my special gratefulness to Associate Prof. Dr. Basman Elhadidi for the great knowledge he gave to me in fluid mechanics through my undergraduate and postgraduate studies. He had always provided me with continuous advices and encouragement that helped in building my academic knowledge.

My parents have always been my first mentors, and my strongest supports through all the obstacles and difficulties I have encountered in my life. I wish to express my special appreciation to my mother for the effort that she had exerted in my early education phases and for her prayers. Also, my father has never hesitated to provide me with the advice and support whenever I needed them. I can not forget my brothers too; Ahmed, Zeyad, Omran, and Gihana for their help and support. Finally, my kind acknowledgment is for my wife, Hadeer, who had been my weal and woe partner through almost tow years of my academic and research work, supporting and encouraging me, and making my home an earth paradise.

# Table of Contents

<b>ACKNOWLEDGEMENTS</b>	<b>i</b>
<b>TABLE OF CONTENTS</b>	<b>ii</b>
<b>LIST OF TABLES</b>	<b>v</b>
<b>LIST OF FIGURES</b>	<b>vi</b>
<b>NOMENCLATURE</b>	<b>ix</b>
<b>ABSTRACT</b>	<b>xii</b>
<b>CHAPTER 1: INTRODUCTION</b>	<b>1</b>
1.1 BACKGROUND . . . . .	1
1.2 LOW ORDER VS HIGH ORDER SCHEMES . . . . .	1
1.3 HIGHER ORDER COMPACT SCHEMES ON UNSTRUCTURED GRIDS	3
1.4 THESIS MAIN OBJECTIVES . . . . .	5
1.5 ORGANIZATION OF THE THESIS . . . . .	5
<b>CHAPTER 2: LITERATURE SURVEY</b>	<b>6</b>
2.1 INTRODUCTION . . . . .	6
2.2 SPECTRAL DIFFERENCE METHOD . . . . .	6
<b>CHAPTER 3: MATHEMATICAL MODELS AND NUMERICAL TREATMENT</b>	<b>9</b>
3.1 NAVIER-STOKES EQUATIONS . . . . .	9
3.2 EULER EQUATIONS . . . . .	11
3.3 GOVERNING EQUATIONS IN A GENERALIZED COORDINATE SYSTEM	12
3.4 INTRODUCTION TO THE SPECTRAL DIFFERENCE METHOD . . . .	13
3.5 TRANSFORMATION OF THE EQUATIONS . . . . .	14
3.6 SPECTRAL DIFFERENCE DISCRETIZATION . . . . .	15

3.6.1	Spectral Difference solution basis polynomials . . . . .	18
3.6.2	Spectral Difference flux basis polynomials . . . . .	18
3.7	CONVECTIVE FLUX VECTOR EVALUATION IN THE SPECTRAL DIFFERENCE METHOD . . . . .	19
3.7.1	Convective interface Flux Treatment . . . . .	20
3.7.1.1	Flux Vector splitting . . . . .	21
3.7.1.2	Flux Difference Splitting . . . . .	23
3.8	DIFFUSIVE FLUX VECTOR EVALUATION IN THE SPECTRAL DIFFERENCE METHOD . . . . .	26
3.8.1	Diffusive interface flux treatment . . . . .	27
3.8.1.1	Bassi and Rebay original approach . . . . .	27
3.9	RESIDUALS COMPUTATION . . . . .	28
3.10	TIME INTEGRATION METHODS . . . . .	28
3.11	SPECTRAL DIFFERENCE ALGORITHM . . . . .	30
3.12	BOUNDARY CONDITIONS . . . . .	31
3.12.1	Inlet/Outlet boundary condition . . . . .	31
3.12.1.1	Subsonic Inlet/Outlet . . . . .	31
3.12.1.2	Supersonic Inlet/Outlet . . . . .	32
3.12.2	Farfield boundary condition . . . . .	32
3.12.3	Inviscid slip wall boundary condition . . . . .	34
3.12.4	Viscous no-slip wall boundary condition . . . . .	34
3.13	GENERAL VALIDATION PARAMETERS FOR NUMERICAL SOLUTIONS	34
3.14	GRID GENERATION . . . . .	36
<b>CHAPTER 4: CODE VALIDATION &amp; TEST CASES</b>		<b>37</b>
4.1	INTRODUCTION . . . . .	37
4.2	QUASI ONE-DIMENSIONAL SUPERSONIC FLOW INSIDE A DIVERGING NOZZLE CONFIGURATION . . . . .	37
4.3	INVISCID FLOW SOLVER . . . . .	42
4.3.1	Subsonic Internal Flow inside a Channel with a ramp . . . . .	42
4.3.2	Subsonic Flow over a Circular Cylinder . . . . .	48
4.3.3	Subsonic Flow over a NACA0012 Airfoil . . . . .	57

4.4	VISCOUS FLOW SOLVER . . . . .	64
4.4.1	Laminar Flow over a Circular Cylinder . . . . .	65
4.4.1.1	Steady Flow Case . . . . .	67
4.4.1.2	Unsteady Flow Case . . . . .	71
4.5	FLOW OVER AN AIRFOIL–SPOILER CONFIGURATION . . . . .	77
4.5.1	Problem Definition . . . . .	78
4.5.2	Results and Discussion . . . . .	82
4.5.2.1	Time averaged pressure distribution, and aerodynamic force coefficients . . . . .	82
4.5.2.2	Time history of Aerodynamic force coefficients . . . . .	86
4.5.2.3	Vortex shedding structure . . . . .	87
<b>CHAPTER 5: CONCLUSIONS, AND FUTURE RECOMMENDATIONS</b>		<b>92</b>
5.1	CONCLUSIONS . . . . .	92
5.2	FUTURE RECOMMENDATIONS AND PENDING ITEMS . . . . .	93
<b>APPENDIX A: GOVERNING EQUATIONS TRANSFORMATION</b>		<b>94</b>
<b>APPENDIX B: ISOPARAMETRIC MAPPING</b>		<b>97</b>
B.1	BILINEAR MAPPING . . . . .	97
B.2	BIQUADRATIC MAPPING . . . . .	98
B.3	BICUBIC MAPPING . . . . .	98
B.4	PROCEDURE TO COMPUTE THE COEFFICIENTS OF THE MAPPING FUNCTIONS $\varphi_K$ FOR ARBITRARY MAPPING ORDER . . . . .	99
<b>APPENDIX C: LEGENDRE GAUSS QUADRATURE POINTS</b>		<b>100</b>
<b>REFERENCES</b>		<b>101</b>



# List of Tables

3.1	1D Legendre Gauss quadrature points for $(p = 2), 3^{rd}$ order SD scheme . . .	16
4.1	h-refinement convergence analysis for the 1D isentropic flow inside a nozzle .....	40
4.2	Aerodynamic coefficients for Subsonic flow over NACA0012 airfoil , $M = 0.63, \alpha = 2^\circ$ , Im: no. of points on the airfoil surface . . . . .	61
4.3	Aerodynamic data of the steady flow over a circular cylinder at $Re = 40$ , and comparison with numerical and experimental data . . . . .	69
4.4	Aerodynamic data of the unsteady flow past a circular cylinder at $Re = 100$ , and comparison with numerical and experimental data . . . . .	72
B.1	Coordinates of the bilinear element nodes in the standard element .....	97
B.2	Coordinates of the biquadratic element nodes in the standard element .....	98
B.3	Coordinates of the bicubic element nodes in the standard element .....	98
C.1	Roots of the higher degree Legendre polynomials $P_n(\xi)$ .....	100

# List of Figures

3.1	SD spatial discretization on 1D cells, (●) solution points, (■) -flux points	15
3.2	SD spatial discretization on 2D quadrilateral cell with component wise flux distribution. (●) solution points, (■) $\xi$ -flux points, (▲) $\eta$ -flux points	17
4.1	Comparison of M,P for 1D supersonic flow inside a nozzle for various schemes	39
4.2	Total Pressure loss $ \Delta p_t $ equation (3.68) for the 1D supersonic flow inside a nozzle	40
4.3	Entropy Error (h Refinement) for the 1D supersonic flow inside a nozzle, (---) lines are the exact slopes for $2^{nd}$ , $3^{rd}$ , $4^{th}$ order convergence rate	41
4.4	Comparison of the Entropy Error (h and p-Refinement) for the 1D supersonic flow inside a nozzle, $\Delta x = 1 ft$	41
4.5	Channel with a ramp (h refinement) meshes	43
4.6	Clustered meshes entropy Error through (h Refinement) for a channel with a ramp, (---) lines are the exact slopes for $2^{nd}$ , $3^{rd}$ , $4^{th}$ , and $5^{th}$ order convergence rates	44
4.7	Comparison of Error norms between uniform and clustered meshes through(h refinement)for a channel with a ramp	45
4.8	Mach Contours through p refinement for the inviscid subsonic flow inside a channel with a ramp	46
4.9	Comparison between h and p-refinement analysis for the inviscid subsonic flow inside a channel with a ramp starting from the coarsest grid	47
4.10	Convergence history for the inviscid subsonic flow inside a channel with a ramp for the $2^{nd}$ SD scheme and using different Riemann solvers	47
4.11	h-refinement meshes for subsonic flow over a circular cylinder	49
4.12	Mach number contours in h-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ , $2^{nd}$ order SD, linear wall elements	50
4.13	Pressure contours in h-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ , $2^{nd}$ order SD, linear wall elements	51
4.14	Cp distribution in h-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ , $2^{nd}$ order SD, linear wall elements	52
4.15	Total pressure ratio $P_t/P_{t_\infty}$ for inviscid subsonic flow over a circular cylinder $M = 0.38$ , $2^{nd}$ order SD, linear wall elements	52
4.16	Comparison of Mach number contours for inviscid subsonic flow over a circular cylinder $M = 0.38$ , different scheme orders and different wall elements orders, Mesh $64 \times 16$	53
4.17	Total pressure ratio $P_t/P_{t_\infty}$ for inviscid subsonic flow over a circular cylinder $M = 0.38$ , $2^{nd}$ order SD, and different wall elements orders, Mesh $64 \times 16$	54
4.18	Mach number contours through p-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ on the coarsest mesh $16 \times 4$	55
4.19	Cp distribution through p-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ , mesh $16 \times 4$	56
4.20	Total pressure ratio $P_t/P_{t_\infty}$ through p-refinement for inviscid subsonic flow over a circular cylinder $M = 0.38$ , mesh $16 \times 4$	56

4.21	Convergence rate comparison between h- and p-refinements for inviscid subsonic flow over a circular cylinder $M = 0.38$ . . . . .	57
4.22	Grid 80 x 24 with outer radius of 30 chord lengths for subsonic flow over a NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	58
4.23	Close up to the LE curved boundary for bicubic mapping elements of a NACA0012 airfoil, (... ○) bicubic element nodes, (—▲) -mesh nodes . . . . .	60
4.24	Convergence history of $L_2$ norm of the density residual for the NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	61
4.25	Mach number distribution over the surface of the NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	62
4.26	Pressure Coefficient ( $C_p$ ) distribution over the surface of the NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	62
4.27	Total pressure loss $ \Delta P_t $ distribution over the surface of the NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	63
4.28	$C_p$ contours for subsonic flow over a NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ .	63
4.29	Mach number contours for subsonic flow over a NACA0012 airfoil, $M = 0.63, \alpha = 2^\circ$ . . . . .	64
4.30	Computational Grid for viscous flow past a circular cylinder . . . . .	66
4.31	Convergence history of the density residual ( $L_2$ norm) for steady flow past a circular cylinder at $Re = 40$ , Mach no.=0.15, using all SD schemes . . . . .	67
4.32	Streamlines for steady flow past a circular cylinder at $Re = 40$ , and Mach no.=0.15, using 4 <sup>th</sup> order SD and bicubic wall . . . . .	68
4.33	$C_p$ distribution on the cylinder surface for steady flow past a circular cylinder at $Re = 40$ , and Mach no.=0.15, using 4 <sup>th</sup> order SD and bicubic wall . . . . .	68
4.34	$\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ , vorticity distribution on the cylinder surface for steady flow past a circular cylinder at $Re = 40$ , and Mach no.=0.15, using 4 <sup>th</sup> order SD and bicubic wall . . . . .	69
4.35	Mach contours for steady flow past a circular cylinder at $Re = 40$ , and Mach no.=0.15, using 4 <sup>th</sup> order SD and bicubic wall . . . . .	70
4.36	Nondimensional Vorticity magnitude contours ( $\frac{\zeta}{\zeta_{max}} =  \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} $ ), for steady flow past a circular cylinder at $Re = 40$ , and Mach no.=0.15, using 4 <sup>th</sup> order SD and bicubic wall . . . . .	70
4.37	$C_p$ distribution over the cylinder surface for the unsteady flow past a circular cylinder at $Re = 100$ , and $M = 0.2$ , using 4 <sup>th</sup> order SD and bicubic wall . .	72
4.38	Convergence history of the density residual ( $L_2$ norm) for for unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using all SD schemes . . .	73
4.39	Convergence history of aerodynamic force coefficients for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall . . . . .	73
4.40	Periodic change of aerodynamic force coefficients for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall	74

4.41	Fast fourier Transform of the lift coefficient $C_L$ for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall	74
4.42	Instantaneous Mach contours for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall	75
4.43	Instantaneous non-dimensional vorticity magnitude contours ( $\frac{\zeta}{\zeta_{max}} =  \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} $ ), for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall	75
4.44	Streamline periodic pattern for the unsteady flow past a circular cylinder at $Re = 100$ , and $M=0.2$ , using 4 <sup>th</sup> order SD and bicubic wall	76
4.45	Geometry of the BATR airfoil-spoiler configuration	78
4.46	Computational Mesh for the BATR airfoil-spoiler at $\delta = 60^\circ$ , $\alpha = 0^\circ$	79
4.47	Airfoil and wake view of the BATR airfoil-spoiler configuration mesh	80
4.48	spoiler and trailing edge view of the BATR airfoil-spoiler configuration mesh	81
4.49	Cp distribution over BATR airfoil-spoiler at $\delta = 60^\circ$ , $\alpha = 0^\circ$ and comparison with experimental and computational data	83
4.50	Cp distribution over BATR airfoil-spoiler at $\delta = 60^\circ$ , $\alpha = 8^\circ$ and comparison with experimental and computational data	83
4.51	Comparison of Cp distribution for different angles of attack and different spoiler deflection angles for the flow field around BATR airfoil-spoiler configuration	84
4.52	Comparison of the time averaged lift and drag coefficients against $\alpha$ , $\delta$ , for the BATR airfoil-spoiler configuration	85
4.53	Base pressure Coefficient (Cpb) for the BATR airfoil-spoiler configuration and comparison with experimental and computational data	85
4.54	Time history of the lift coefficient ( $C_L$ ) for the BATR airfoil-spoiler at different spoiler deflection angles and zero angle of attack	86
4.55	Fast Fourier transform (FFT) of the lift coefficient ( $C_L$ ) for the BATR airfoil-spoiler at different spoiler deflection angles and zero angle of attack	86
4.56	Strouhl number based on the spoiler projection height for the BATR airfoil-spoiler at $\alpha = 0^\circ$ and comparison with experimental data	87
4.57	Streamline periodic pattern for the BATR airfoil-spoiler at $\delta = 60^\circ$ , $\alpha = 0^\circ$	88
4.58	Streamline periodic pattern for the BATR airfoil-spoiler at $\delta = 60^\circ$ , $\alpha = 8^\circ$	89
4.59	Streamline periodic pattern for the BATR airfoil-spoiler at $\delta = 30^\circ$ , $\alpha = 0^\circ$	90
4.60	Streamline periodic pattern for the BATR airfoil-spoiler at $\delta = 30^\circ$ , $\alpha = 8^\circ$	91
B.1	Bilinear Mapping	97
B.2	Biquadratic Mapping	98

# Nomenclature

$[A_n]$	Normal Jacobian matrix
$\alpha$	Angle of attack
$\varphi(\vec{\xi})$	Shape functions for the isoparametric transformation
$\epsilon_s$	Entropy error
$\gamma$	Specific heat ratio, 1.4 for air
$\mu$	Dynamic viscosity coefficient
$\phi^{fp}$	Lagrange interpolation polynomial for the flux points
$\Phi_l^{fp}(\vec{\xi})$	Flux basis polynomials
$\psi^{sp}$	Lagrange interpolation polynomial for the solution points
$\Psi_i^{sp}(\vec{\xi})$	Solution basis polynomials
$\rho$	Density
$\tau_{xx}, \tau_{xy}, \tau_{yy}$	Shear stress tensor elements
$\vec{F}$	Total flux vector in cell local computational coordinate $\vec{\xi}$
$\vec{F}_c$	Convective flux vector in cell local computational coordinate $\vec{\xi}$
$\vec{F}_d$	Diffusive flux vector in cell local computational coordinate $\vec{\xi}$
$\tilde{f}_c, \tilde{g}_c$	$\xi, \eta$ -components of convective flux vector, respectively, in cell local computational coordinate $\vec{\xi}$
$\tilde{f}_c^{common}, \tilde{g}_c^{common}$	Interface common convective flux components in computational domain
$\tilde{f}_d, \tilde{g}_d$	$\xi, \eta$ -components of diffusive flux vector, respectively, in cell local computational coordinate $\vec{\xi}$
$\tilde{f}, \tilde{g}$	$\xi, \eta$ -components of the total flux vector in cell local computational coordinate $\vec{\xi}$
$\tilde{f}^k(\vec{\xi}), \tilde{g}^k(\vec{\xi})$	Computational flux polynomials in cell $k$
$\tilde{Q}$	conserved variables in cell local computational coordinate $\vec{\xi}$
$\tilde{Q}_f^k, \tilde{Q}_g^k$	Computational conserved variables polynomials defined at $\xi, \eta$ -flux points, respectively for cell $k$
$\vec{\nabla}$	Gradient operator vector in the Cartesian coordinates system: $[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}]^T$

$\vec{\nabla}_{\vec{\xi}}$	Gradient operator vector in the computational coordinates system: $[\frac{\partial}{\partial \xi}, \frac{\partial}{\partial \eta}]^T$
$\vec{\xi}$	Vector of computational coordinates system
$\vec{F}$	Total flux vector in physical domain
$\vec{F}_c$	Convective flux vector in physical domain
$\vec{F}_d$	Diffusive flux vector in physical domain
$\vec{n}$	Unit normal vector in the physical Cartesian coordinates system
$\vec{x}$	Vector of Cartesian coordinates system
$\vec{\nabla} Q^k \approx \vec{\Theta}(\vec{\xi})$	Conserved variables gradient polynomial
$\widetilde{Res}_i(\vec{\xi}^{sp})$	Residual of the governing equations in computational domain
$\xi, \eta$	Computational coordinates
$\xi^{fp}$	Flux points coordinates in 1D
$\xi^{sp}$	Solution points coordinates in 1D
$\xi_x, \xi_y, \eta_x, \eta_y$	Metrics of the transformation
$c$	Speed of sound
$C_D$	Drag coefficient
$C_L$	Lift coefficient
$C_p$	Pressure Coefficient
$E_{c,n}$	Interface normal convective flux component in physical Cartesian coordinate system
$e_t$	Specific total energy
$f$	x-component of the total flux vector in physical Cartesian coordinates
$f_c, f_d$	x-components of the convective and diffusive flux vectors, respectively, in physical Cartesian coordinates
$f_r$	Frequency in hertz
$g$	y-component of the total flux vector in physical Cartesian coordinates
$g_c, g_d$	y-components of the convective and diffusive flux vectors, respectively, in physical Cartesian coordinates
$J$	Jacobian of the transformation
$L_c$	Characteristic length

$M_\infty$	Free stream Mach number
$N$	Number of solution point in 1D cells
$N^{fp}$	Number of flux points in d-dimensional elements
$N^{sp}$	Number of solution points in d-dimensional elements
$n_x, n_y$	Components of the physical unit normal vector $\vec{n}$
$Nn$	Total number of geometric nodes/vertices used to define the physical element
$P$	Pressure
$p$	Polynomial order
$P_t$	Total pressure
$Pr$	Prandtl number, 0.72 for air
$Q$	Conserved variables in physical domain
$Q^k(\vec{\xi}), \tilde{Q}^k(\vec{\xi})$	Physical and computational solution polynomials in cell $k$ , respectively
$q_x, q_y$	Heat flux vector components in x,y directions, respectively
$R_x, R_y$	Total forces in x,y-directions, respectively
$Re_v$	Reynolds number
$Rn$	One-dimensional Riemann invariants
$s$	Entropy
$S_A$	Area of the nozzle for the quasi-one dimensional nozzle flow
$S_t$	Strouhl number
$T$	Temperature
$t$	Time
$T_t$	Total temperature
$u, v$	Velocity components in x,y directions, respectively, $m/s$
$V_\infty$	Free stream velocity
$V_n$	Normal velocity component in the physical Cartesian coordinate system
$V_t$	Tangent velocity component in the physical Cartesian coordinate system
$x, y$	Cartesian coordinates