Ain Shams University

On Difference Cordial Graphs and Other Graphs

Thesis by

Shakir Mahmoud Salman Al-Azzawy

Submitted to

Department of mathematics - Faculty of Science

Ain Shams University - Cairo - Egypt

for the Degree of Doctor of Philosophy

in Pure Mathematics

Supervisors

Prof. Dr. Mohammed Abdel Azim Seoud

Emeritus Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Dr. Labib Rashed El-Sayed Awad

Assistant Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Cairo 2016

On Difference Cordial Graphs and Other Graphs

Thesis by

Shakir Mahmoud Salman Al-Azzawy

Submitted To

Department of Mathematics-Faculty of Science

Ain Shams University

for the Degree of Doctor of Philosophy

in Pure Mathematics

Supervisors

Prof. Dr. Mohammed Abdel Azim Seoud

Emeritus Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Dr. Labib Rashed El-Sayed Awad

Assistant Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Cairo 2016

APPROVAL SHEET

Name: Shakir Mahmoud Salman Al-Azzawy

Title: On Difference Cordial Graph and Other Graphs

Supervised By

Prof. Dr. Mohammed Abedel Azim Seoud

Emeritus Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Dr. Labib Rashid El-Sayed Awad

Associate Professor of Pure Mathematics

Department of Mathematics-Faculty of Science

Ain Shams University

Date / / 2016

Page of Title

Name: Shakir Mahmoud Salman Al-Azzawy

Degree: Doctor of Philosophy in Pure Mathematics

Department: Mathematics

Faculty: Science

University: Ain Shams

Graduation Date: / / 2016

Registration: / / 2012

Grant Date: / / 2017

Cairo - 2017

ACKNOWLEDGMENT

In the name of Allah, the most merciful, the most compassionate all praise be to Allah, the Lord of the worlds; and prayer and peace be upon his servant and messenger Mohamed alayhi wa-alehe wa-sallam.
First of all gratitude and thanks to gracious ***Allah*** *who always helps and guides me. I would like to thank* ***the prophet Mohamed*** *"peace be upon him" who urges us to seek knowledge and who is the teacher of mankind.*

I wish to express my deepest gratitude and thankfulness to my supervisor Professor **Mohammed Abd El-Azim Seoud** *for his invaluable suggestions, continuous encouragement, constant support, guidance and constructive criticism during the period of witting the thesis, Im lucky to have an advisor who works out of genuine curiosity and a passion for research, and who knows how to have a measure of fun in doing it. It is hard to find words to thank him for all the ways he has helped me. Also, I wish to express my great thanks for Assistant Professor Labib Rashed and I wish to express my great thanks to the chairman and staff of the Department of Mathematics, Faculty of Science, Ain Shams University, for their kind assistance and facilities offered through this investigation.*

Also, I would like to express my sincere thanks and deepest gratitude to my family for their patience throughout the preparation of this thesis.

Shakir Al-Azzawy

Abstract

Graph labeling is one of the important branches of Graph Theory and became a principal tool in many applications on different sciences and technologies. All that leads to appearance of more than one type of labeling and multiple techniques to meet the required purposes.

In this thesis we study the two main types of graph labeling and introduce the labelings for interested families of graphs and a tractive results for graphs of these types. We state some basic definitions and theorems in graph theory which we need. We divide the other work into four chapters:

In chapter two we introduce some results in difference cordial graphs and difference cordial labelings for some families of graphs such as: ladder, triangular ladder, grid, step ladder and two sided step ladder graph. Also we discussed some families of graphs which may be difference cordial or not, such as diagonal ladder and some types of one-point union of graphs.

In chapter three we introduce some results on difference cordial graphs, where we present results concerning the relation between difference cordiality and the lengths of paths on graphs and study the Semi-Hamiltonian graph, biconnected outerplanar graphs and the line graph of

a graph. Also, we describe the difference cordial labeling for some families of graphs such as: the graph obtained by duplication a vertex by an edge, bow graphs, butterfly graphs, shell-flower graphs and one-point union of complete graphs.

In chapter four we introduce some results on divisor cordial graphs and describe the divisor cordial labeling for the families of graphs: the jelly fish graph, the shell, the bow graph, butterfly graphs and the friendship graphs. In the last chapter we introduce results in divisor cordial labeling for regular graphs, divisor labelings for all graphs with number of vertices less than eight, and divisor cordial labelings for some types of trees such as: olive trees, spider trees, $m-$star trees, $k-$distant trees, caterpillar trees and banana trees.

Contents

ACKNOWLEDGMENT vi

Abstract vii

List of Figures xii

Summary 1

Contents 1

1 Introduction 4

1.1 Brief Introduction to Labeling 4

1.2 Some Fundamentals in Graph Theory: 8

1.2.1 Some Types of Graphs 12

1.2.2 Operations on Graphs 15

2 On Difference Cordial Graphs 17

2.1 Introduction 17

2.2 Main Results 19

2.3 Difference cordial labeling for Some graphs 25

2.3.1 Ladder graphs L_n . . . 25
2.3.2 Triangular ladder graph TL_n . . . 31
2.3.3 The Grid graph $P_m \times P_n$ . . . 32
2.3.4 Step ladder graph $S(T_n)$: . . . 37
2.3.5 Double Sided Step Ladder Graph $2S(T_{2n})$: . . . 39

3 Some Results and Examples on Difference Cordial Graphs 51

3.1 Introduction . . . 51
3.2 Some Results . . . 53
3.3 Difference Cordial Labeling for Some Families of Graphs . 59
3.3.1 Graph Obtained by Duplication of Vertex by an Edge 59
3.3.2 Bow Graphs . . . 61
3.3.3 Butterfly Graphs . . . 62
3.3.4 Shell-Flower Graphs . . . 63
3.3.5 One-Point Union of Complete Graphs . . . 65

4 Some Results on Divisor Cordial Graphs 68

4.1 Introduction . . . 68
4.2 The Results . . . 70
4.3 Divisor Cordial Labeling for Some Families of Graphs . . . 72
4.3.1 The Jelly Fish Graph . . . 72
4.3.2 The shell and The Bow Graph . . . 74
4.3.3 Butterfly Graphs . . . 78
4.3.4 Friendship Graphs . . . 79

5 Divisor Cordial Labeling for Some Trees and Families of Graphs **81**

5.1 Introduction 81

5.2 The Results 83

5.3 Divisor Cordial Labeling for Some Trees 85

5.3.1 Olive Tree 86

5.3.2 Spider Tree 88

5.3.3 m-stars Tree 88

5.3.4 k-distant tree 90

5.3.5 Caterpillar Tree 90

5.3.6 Banana Tree 91

A All Nonisomorphic Graphs with 7 Vertices and its Divisor Cordial Graphs **98**